

## Chapter 6

### Example of Calculation Procedure

#### 1. ESC TEST

##### 1.1. Gaseous emissions

The measurement data for the calculation of the individual mode results are shown below. In this example, CO and NO<sub>x</sub> are measured on a dry basis, HC on a wet basis. The HC concentration is given in propane equivalent (C3) and has to be multiplied by 3 to result in the C1 equivalent. The calculation procedure is identical for the other modes.

P(kW)	82.9
T <sub>a</sub> (K)	294.8
H <sub>a</sub> (g/Kg)	7.81
q <sub>mew</sub> (Kg/h)	563.38
q <sub>maw</sub> (Kg/h)	545.29
q <sub>mf</sub> (Kg/h)	18.09
HC (ppm)	6.3
CO (ppm)	41.2
NO <sub>x</sub> (ppm)	495
W <sub>ALF</sub> , % mass	15.38
W <sub>BET</sub> , % mass	84.60
W <sub>GAM</sub> , % mass	0.005
W <sub>DEL</sub> , % mass	0.011
W <sub>EPS</sub> , % mass	0.004

Calculation of the dry to wet correction factor K<sub>w,r</sub> (Chapter III, Appendix 1, Section 5.2)

$$k_{w,r} = \left( 1 - \frac{1,2442 \times H_a + 111,19 \times w_{ALF} \times \frac{q_{mf}}{q_{mad}}}{773,4 + 1,2442 \times H_a + \frac{q_{mf}}{q_{mad}} \times k_f \times 1000} \right) \times 1,008$$

$$q_{mad} = q_{maw} / k_{w,a}$$

$$k_{w,a} = 1 - \frac{(1.608 \times 7.81)}{1000 + (1.608 \times 7.81)} = 0.9876$$

$$q_{mad} = 545.29 / 0.9876 = 552.138$$

$$k_f = 0,055584 \times 15.38 - 0,0001083 \times 84.60 - 0,0001562 \times 0.005 + 0,0079936 \times 0.011 + 0,0069978 \times 0.004 = 0.8458$$

$$k_{w,r} = 0.9263$$

Calculation of the wet concentrations:

$$\text{CO} = 41,2 \times 0,9263 = 38,16 \text{ ppm}$$

$$\text{NO}_x = 495 \times 0,9263 = 458.5 \text{ ppm}$$

Calculation of the NO<sub>x</sub> humidity correction factor  $K_{h,D}$  (Chapter III, appendix 1, section 5.3):

$$k_{h,D} = \frac{1}{1 - 0,0182 \times (H_a - 10,71) + 0,0045 \times (T_a - 298)}$$

$$= 0.9630$$

Calculation of the emission mass flow rates (Chapter III, appendix 1, section 5.4):

$$\text{NO}_x = 0,001587 \times 458.5 \times 0,9630 \times 563,38 = 394.78 \text{ g/h}$$

$$\text{CO} = 0,000966 \times 38,16 \times 563,38 = 20,767 \text{ g/h}$$

$$\text{HC} = 0,000479 \times 6,3 \times 3 \times 563,38 = 5,100 \text{ g/h}$$

Calculation of the specific emissions (Chapter III, appendix 1, section 5.5):

The following example calculation is given for CO; the calculation procedure is identical for the other components.

The emission mass flow rates of the individual modes are multiplied by the respective weighting factors, as indicated in Chapter III, appendix 1, section 2.7.1, and summed up to result in the mean emission mass flow rate over the cycle:

$$\begin{aligned} \text{CO} = & (6,7 \times 0,15) + (24,6 \times 0,08) + (20,5 \times 0,10) + (20,7 \times 0,10) + (20,6 \times 0,05) + \\ & (15,0 \times 0,05) + (19,7 \times 0,05) + (74,5 \times 0,09) + (31,5 \times 0,10) + (81,9 \times 0,08) \\ & + (34,8 \times 0,05) + (30,8 \times 0,05) + (27,3 \times 0,05) \\ & = 30,91 \text{ g/h} \end{aligned}$$

The engine power of the individual modes is multiplied by the respective weighting factors, as indicated in Chapter III, appendix 1, section 2.7.1, and summed up to result in the mean cycle power:

$$\begin{aligned} P(n) = & (0,1 \times 0,15) + (96,8 \times 0,08) + (55,2 \times 0,10) + (82,9 \times 0,10) + (46,8 \times 0,05) \\ & + (70,1 \times 0,05) + (23,0 \times 0,05) + (114,3 \times 0,09) + (27,0 \times 0,10) + (122,0 \times \end{aligned}$$

$$0,08) + (28,6 \times 0,05) + (87,4 \times 0,05) + (57,9 \times 0,05)$$

$$= 60,006 \text{ kW}$$

$$\text{CO} = 30,91 / 60,006 = 0,0515 \text{ g / kWh}$$

Calculation of the specific NO<sub>x</sub> emission of the random point (Chapter III, appendix 1, Section 5.6.1):

Assume the following values have been determined on the random point:

$$n_z = 1\,600 \text{ min}^{-1}$$

$$M_z = 495 \text{ Nm}$$

$$\text{NO}_{x \text{ mass},z} = 487,9 \text{ g/h (calculated according to the previous formulae)}$$

$$P(n)_z = 83 \text{ kW}$$

$$\text{NO}_{x,z} = 487,9/83 = 5,878 \text{ g/kWh}$$

Determination of the emission value from the test cycle (Chapter III, appendix 1, section 5.6.2):

Assume the values of the four enveloping modes on the ESC to be as follows:

n <sub>RT</sub>	n <sub>SU</sub>	E <sub>R</sub>	E <sub>S</sub>	E <sub>T</sub>	E <sub>U</sub>	M <sub>R</sub>	M <sub>S</sub>	M <sub>T</sub>	M <sub>U</sub>
1 368	1 785	5,943	5,565	5,889	4,973	515	460	681	610

$$E_{TU} = 5,889 + (4,973 - 5,889) \times (1\,600 - 1\,368) / (1\,785 - 1\,368) = 5,377 \text{ g/kWh}$$

$$E_{RS} = 5,943 + (5,565 - 5,943) \times (1\,600 - 1\,368) / (1\,785 - 1\,368) = 5,732 \text{ g/kWh}$$

$$M_{TU} = 681 + (601 - 681) \times (1\,600 - 1\,368) / (1\,785 - 1\,368) = 641,3 \text{ Nm}$$

$$M_{RS} = 515 + (460 - 515) \times (1\,600 - 1\,368) / (1\,785 - 1\,368) = 484,3 \text{ Nm}$$

$$E_z = 5,732 + (5,377 - 5,732) \times (495 - 484,3) / (641,3 - 484,3) = 5,708 \text{ g/kWh}$$

Comparison of the NO<sub>x</sub> emission values (Chapter III, appendix 1, Section 5.6.3):

$$\text{NO}_{x \text{ diff}} = 100 \times (5,878 - 5,708) / 5,708 = 2,98 \%$$

## 1.2. Particulate emissions

Particulate measurement is based on the principle of sampling the particulates over the complete cycle, but determining the sample and flow rates ( $m_{sep}$  and  $q_{medf}$ ) during the individual modes. The calculation of  $q_{medf}$  depends on the system used. In the following examples, a system with CO<sub>2</sub> measurement and carbon balance method and a system with flow measurement are used. When using a full flow dilution system,  $q_{medf}$  is directly measured by the CVS equipment.

Calculation of  $q_{medf}$  (Chapter III, Appendix 1, Sections 5.2.3 and 5.2.4):

Assume the following measurement data of mode 4. The calculation procedure is identical for the other modes.

$q_{mew}$ (Kg/h)	$q_{mf}$ (Kg/h)	$q_{mdw}$ (Kg/h)	$q_{mdew}$ (Kg/h)	CO <sub>2D</sub> (%)	CO <sub>2A</sub> (%)
334.02	10.76	5.4435	6.0	0.657	0.040

(a) carbon balance method

$$q_{medf} = \frac{206,5 \times 10,76}{0,657 - 0,040} = 3\ 601,2 \text{ kg/h}$$

(b) flow measurement method

$$q = 6.0 / (6.0 - 5.4435) = 10.78$$

$$q_{medf} = 334.02 \times 10.78 = 3600.7 \text{ Kg/h}$$

Calculation of the mass flow rate (Chapter III, Appendix 1, Section 5.4):

The  $q_{medf}$  flow rates of the individual modes are multiplied by the respective weighting factors, as indicated in chapter III, Appendix 1, Section 2.7.1, and summed up to result in the mean  $q_{medf}$  over the cycle. The total sample rate  $m_{sep}$  is summed up from the sample rates of the individual modes.

$$\begin{aligned} \bar{q}_{medf} &= (3\ 567 \times 0,15) + (3\ 592 \times 0,08) + (3\ 611 \times 0,10) + (3\ 600 \times 0,10) + \\ & (3\ 618 \times 0,05) + (3\ 600 \times 0,05) + (3\ 640 \times 0,05) + (3\ 614 \times 0,09) + \\ & (3\ 620 \times 0,10) + (3\ 601 \times 0,08) + (3\ 639 \times 0,05) + (3\ 582 \times 0,05) + \\ & (3\ 635 \times 0,05) \\ &= 3\ 604,6 \text{ kg/h} \end{aligned}$$

$$\begin{aligned}
m_{\text{sep}} &= 0,226 + 0,122 + 0,151 + 0,152 + 0,076 + 0,076 + 0,076 + 0,136 + \\
& 0,151 + 0,121 + 0,076 + 0,076 + 0,075 \\
&= 1,515 \text{ kg}
\end{aligned}$$

Assume the particulate mass on the filters to be 2,5 mg, then

$$PT_{\text{mass}} = (2.5 \times 360.4) / (1.515 \times 1000) = 5.948 \text{ g/h}$$

Background correction (optional)

Assume one background measurement with the following values. The calculation of the dilution factor D is identical to Section 3.1 of this chapter and not shown here.

$$m_{f,d} = 0,1 \text{ mg}; m_d = 1,5 \text{ kg}$$

$$\begin{aligned}
\text{Sum of D} &= [(1-1/119,15) \times 0,15] + [(1-1/8,89) \times 0,08] + [(1-1/14,75) \times 0,10] + \\
& [(1-1/10,10) \times 0,10] + [(1-1/18,02) \times 0,05] + [(1-1/12,33) \times 0,05] + [(1-1/32,18) \times 0,05] + \\
& [(1-1/6,94) \times 0,09] + [(1-1/25,19) \times 0,10] + [(1-1/6,12) \times 0,08] + [(1-1/20,87) \times 0,05] + [(1-1/8,77) \times 0,05] + [(1-1/12,59) \times 0,05] \\
&= 0,923
\end{aligned}$$

$$PT_{\text{mass}} = 2.5 / 1.515 - (0.1 / 1.5 \times 0.923) \times 3604.6 / 1000 = 5.726 \text{ g/h.}$$

Calculation of the specific emission (Chapter III, Appendix 1, Section 5.5):

$$\begin{aligned}
P(n) &= (0,1 \times 0,15) + (96,8 \times 0,08) + (55,2 \times 0,10) + (82,9 \times 0,10) + (46,8 \times 0,05) \\
&+ (70,1 \times 0,05) + (23,0 \times 0,05) + (114,3 \times 0,09) + (27,0 \times 0,10) + (122,0 \times 0,08) \\
&+ (28,6 \times 0,05) + (87,4 \times 0,05) + (57,9 \times 0,05) \\
&= 60,006 \text{ kW}
\end{aligned}$$

$$PT = 5.948 / 60.006 = 0,099 \text{ g/kWh}$$

$$PT = (5.726/60.006) = 0,095 \text{ g/kWh, if background corrected}$$

Calculation of the specific weighting factor (Chapter III, Appendix 1, Section 5.6):

Assume the values calculated for mode 4 above, then

$$WF_{E,i} = (0,152 \times 3\,604,6 / 1,515 \times 3\,600,7) = 0,1004$$

This value is within the required value of  $0,10 \pm 0,003$ .

## 2. ELR TEST

Since Bessel filtering is a completely new averaging procedure in European exhaust legislation, an explanation of the Bessel filter, an example of the design of a Bessel algorithm, and an example of the calculation of the final smoke value is given below. The constants of the Bessel algorithm only depend on the design of the opacimeter and the sampling rate of the data acquisition system. It is recommended that the opacimeter manufacturer provide the final Bessel filter constants for different sampling rates and that the customer use these constants for designing the Bessel algorithm and for calculating the smoke values.

### 2.1. General remarks on the Bessel filter

Due to high frequency distortions, the raw opacity signal usually shows a highly scattered trace. To remove these high frequency distortions a Bessel filter is required for the ELR-test. The Bessel filter itself is a recursive, second-order low-pass filter which guarantees the fastest signal rise without overshoot.

Assuming a real time raw exhaust plume in the exhaust tube, each opacimeter shows a delayed and differently measured opacity trace. The delay and the magnitude of the measured opacity trace is primarily dependent on the geometry of the measuring chamber of the opacimeter, including the exhaust sample lines, and on the time needed for processing the signal in the electronics of the opacimeter. The values that characterize these two effects are called the physical and the electrical response time, which represent an individual filter for each type of opacimeter.

The goal of applying a Bessel filter is to guarantee a uniform overall filter characteristic of the whole opacimeter system, consisting of:

- physical response time of the opacimeter ( $t_p$ ),
- electrical response time of the opacimeter ( $t_e$ ),
- filter response time of the applied Bessel filter ( $t_F$ ).

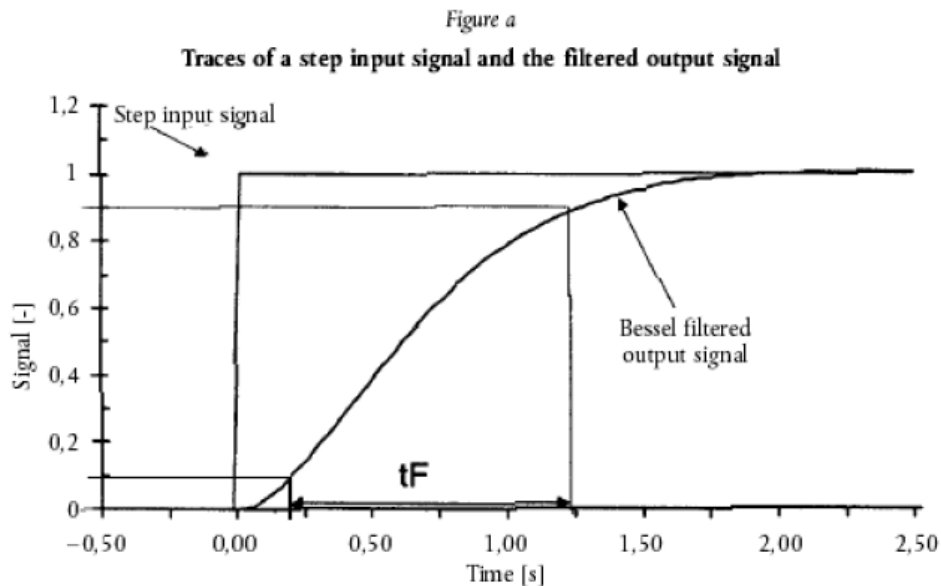
The resulting overall response time of the system  $t_{Aver}$  is given by:

$$t_{Aver} = \sqrt{t_F^2 + t_p^2 + t_e^2}$$

and must be equal for all kinds of opacimeters in order to give the same smoke value. Therefore, a Bessel filter has to be created in such a way, that the filter response time ( $t_F$ ) together with the physical ( $t_p$ ) and electrical response time ( $t_e$ ) of the individual opacimeter must result in the required overall response time ( $t_{Aver}$ ). Since  $t_p$  and  $t_e$  are given values for each individual opacimeter, and  $t_{Aver}$  is defined to be 1,0 s in this Directive,  $t_F$  can be calculated as follows:

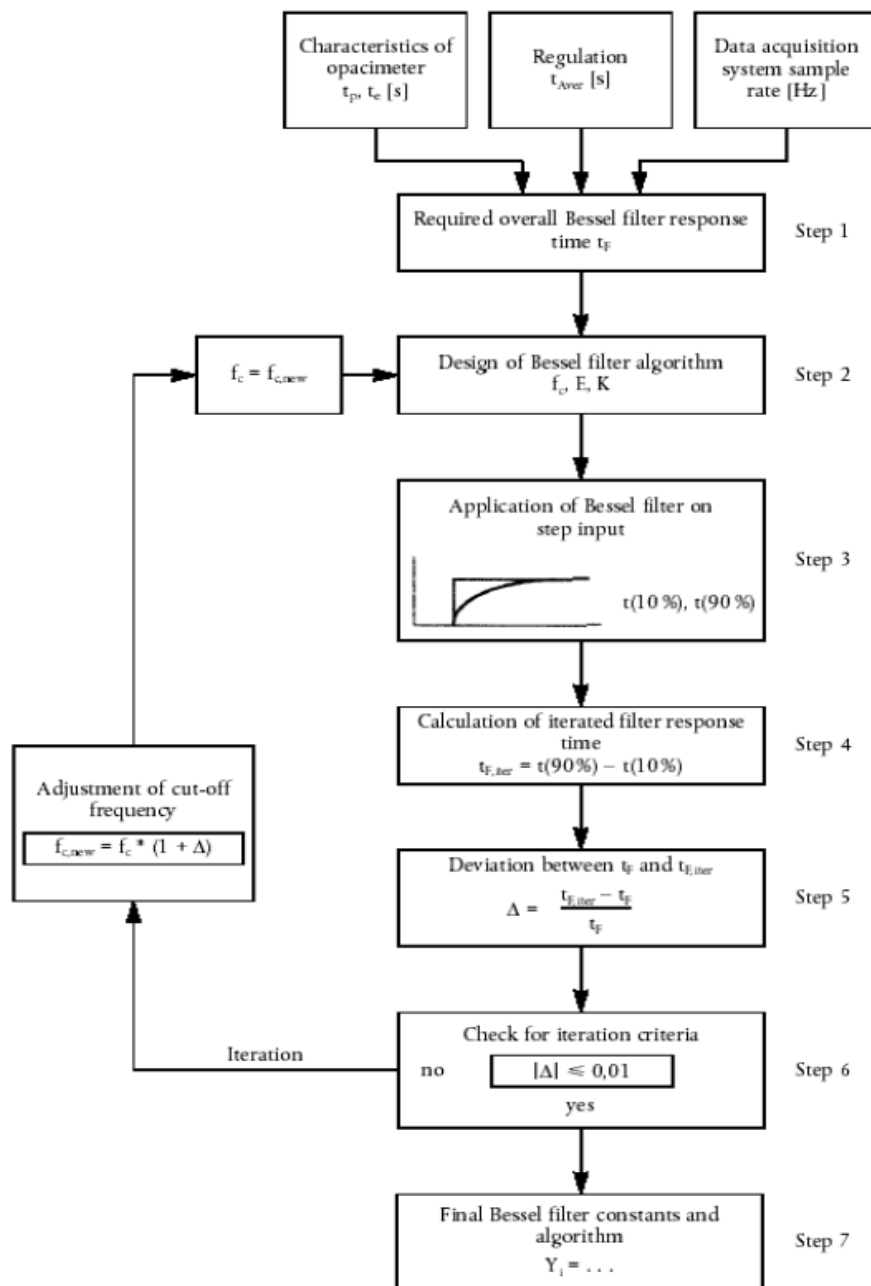
$$t_F = \sqrt{t_{Aver}^2 + t_p^2 + t_e^2}$$

By definition, the filter response time  $t_F$  is the rise time of a filtered output signal between 10 % and 90 % on a step input signal. Therefore the cut-off frequency of the Bessel filter has to be iterated in such a way, that the response time of the Bessel filter fits into the required rise time.



In Figure a, the traces of a step input signal and Bessel filtered output signal as well as the response time of the Bessel filter ( $t_F$ ) are shown.

Designing the final Bessel filter algorithm is a multi step process which requires several iteration cycles. The scheme of the iteration procedure is presented below.





## 2.2. Calculation of the Bessel algorithm

In this example a Bessel algorithm is designed in several steps according to the above iteration procedure which is based upon Chapter III, Appendix 1, Section 6.1.

For the opacimeter and the data acquisition system, the following characteristics are assumed:

- physical response time  $t_p$  0,15 s
- electrical response time  $t_e$  0,05 s
- overall response time  $t_{Aver}$  1,00 s (by definition in this Part)
- sampling rate 150 Hz

**Step 1** Required Bessel filter response time  $t_F$ :

$$t_F = \sqrt{1^2 - (0,15^2 + 0,05^2)} = 0,987421 \text{ s}$$

**Step 2** Estimation of cut-off frequency and calculation of Bessel constants E, K for first iteration:

$$f_c = \frac{3,1415}{10 \times 0,987421} = 0,318152 \text{ Hz}$$

$$\Delta t = 1/150 = 0,006667 \text{ s}$$

$$\Omega = \frac{1}{\tan [3,1415 \times 0,006667 \times 0,318152]} = 150,07664$$

$$E = \frac{1}{1 + 150,076644 \times \sqrt{3} \times 0,618034 + 0,618034 + 150,076644^2} = 7,07948 \times 10^{-5}$$

$$K = 2 \times 7,07948 \times 10^{-5} \times (0,618034 \times 150,076644^2 - 1) - 1 = 0,970783$$

This gives the Bessel algorithm:

$$Y_i = Y_{i-1} + 7,07948 E - 5 \times (S_i + 2 \times S_{i-1} + S_{i-2} - 4 \times Y_{i-2}) + 0,970783 \times (Y_{i-1} - Y_{i-2})$$

where  $S_i$  represents the values of the step input signal (either '0' or '1') and  $Y_i$  represents the filtered values of the output signal.

**Step 3** Application of Bessel filter on step input:

The Bessel filter response time  $t_F$  is defined as the rise time of the filtered output signal between 10 % and 90 % on a step input signal. For determining the times

of 10 % ( $t_{10}$ ) and 90 % ( $t_{90}$ ) of the output signal, a Bessel filter has to be applied to a step input using the above values of  $f_c$ , E and K.

The index numbers, the time and the values of a step input signal and the resulting values of the filtered output signal for the first and the second iteration are shown in Table B. The points adjacent to  $t_{10}$  and  $t_{90}$  are marked in bold numbers.

In Table B, first iteration, the 10 % value occurs between index number 30 and 31 and the 90 % value occurs between index number 191 and 192. For the calculation of  $t_{F,iter}$  the exact  $t_{10}$  and  $t_{90}$  values are determined by linear interpolation between the adjacent measuring points, as follows:

$$t_{10} = t_{lower} + \Delta t \times (0,1 - out_{lower}) / (out_{upper} - out_{lower})$$

$$t_{90} = t_{lower} + \Delta t \times (0,9 - out_{lower}) / (out_{upper} - out_{lower})$$

where  $out_{upper}$  and  $out_{lower}$ , respectively, are the adjacent points of the Bessel filtered output signal, and  $t_{lower}$  is the time of the adjacent time point, as indicated in Table B.

$$t_{10} = 0,200000 + 0,006667 \times (0,1 - 0,099208) / (0,104794 - 0,099208) = 0,200945 \text{ s}$$

$$t_{90} = 0,273333 + 0,006667 \times (0,9 - 0,899147) / (0,901168 - 0,899147) = 1,276147 \text{ s}$$

**Step 4** Filter response time of first iteration cycle:

$$t_{F,iter} = 1,276147 - 0,200945 = 1,075202 \text{ s}$$

**Step 5** Deviation between required and obtained filter response time of first iteration cycle:

$$\Delta = (1,075202 - 0,987421) / 0,987421 = 0,081641$$

**Step 6** Checking the iteration criteria:

$|\Delta| \leq 0,01$  is required. Since  $0,081641 > 0,01$ , the iteration criteria is not met and a further iteration cycle has to be started. For this iteration cycle, a new cut-off frequency is calculated from  $f_c$  and  $\Delta$  as follows:

$$f_{c,new} = 0,318152 \times (1 + 0,081641) = 0,344126 \text{ Hz}$$

This new cut-off frequency is used in the second iteration cycle, starting at step 2 again. The iteration has to be repeated until the iteration criteria is met. The resulting values of the first and second iteration are summarised in Table A

Table A

Values of the first and second iteration

Parameter		1. Iteration	2. Iteration
$f_c$	(Hz)	0,318152	0,344126
E	(-)	7,07948 E-5	8,272777 E-5
K	(-)	0,970783	0,968410
$t_{10}$	(s)	0,200945	0,185523
$t_{90}$	(s)	1,276147	1,179562
$t_{F,iter}$	(s)	1,075202	0,994039
$\Delta$	(-)	0,081641	0,006657
$f_{c,new}$	(Hz)	0,344126	0,346417

**Step 7** Final Bessel algorithm:

As soon as the iteration criteria has been met, the final Bessel filter constants and the final Bessel algorithm are calculated according to step 2. In this example, the iteration criteria has been met after the second iteration ( $\Delta = 0,006657 \leq 0,01$ ). The final algorithm is then used for determining the averaged smoke values (see next Section 2.3).

$$Y_i = Y_{i-1} + 8,272777 \times 10^{-5} \times (S_i + 2 \times S_{i-1} + S_{i-2} - 4 \times Y_{i-2}) + 0,968410 \times (Y_{i-1} - Y_{i-2})$$

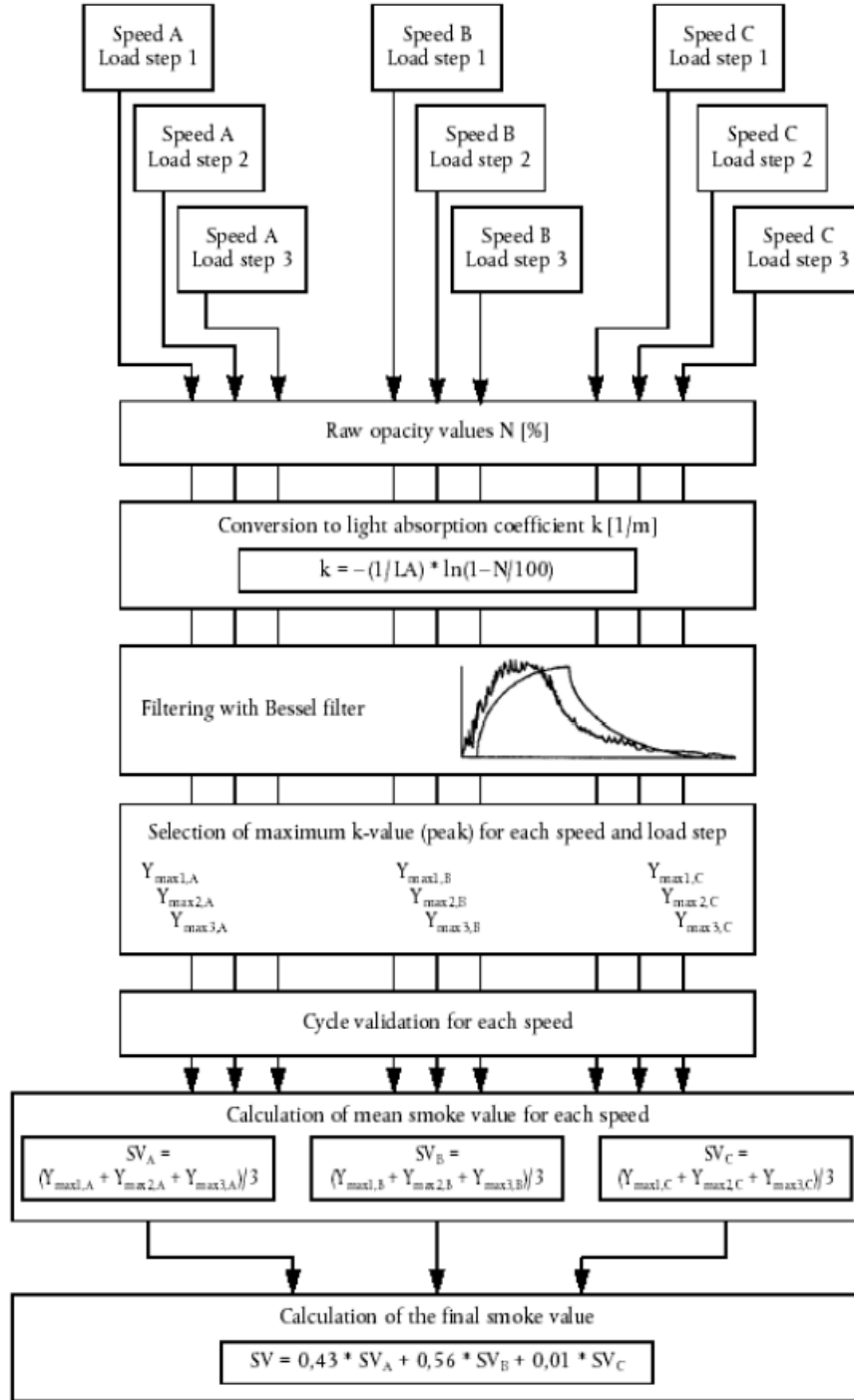
Table B

Values of step input signal and Bessel filtered output signal for the first and second iteration cycle

Index i [-]	Time [s]	Step Input Signal $S_i$ [-]	Filtered Output Signal $Y_i$ [-]	
			1. Iteration	2. Iteration
-2	-0,013333	0	0,000000	0,000000
-1	-0,006667	0	0,000000	0,000000
0	0,000000	1	0,000071	0,000083
1	0,006667	1	0,000352	0,000411
2	0,013333	1	0,000908	0,001060
3	0,020000	1	0,001731	0,002019
4	0,026667	1	0,002813	0,003278
5	0,033333	1	0,004145	0,004828
~	~	~	~	~
24	0,160000	1	0,067877	0,077876
25	0,166667	1	0,072816	0,083476
26	0,173333	1	0,077874	0,089205
27	0,180000	1	0,083047	0,095056
28	0,186667	1	0,088331	0,101024
29	0,193333	1	0,093719	0,107102
30	0,200000	1	0,099208	0,113286
31	0,206667	1	0,104794	0,119570
32	0,213333	1	0,110471	0,125949
33	0,220000	1	0,116236	0,132418
34	0,226667	1	0,122085	0,138972
35	0,233333	1	0,128013	0,145605
36	0,240000	1	0,134016	0,152314
37	0,246667	1	0,140091	0,159094
~	~	~	~	~
175	1,166667	1	0,862416	0,895701
176	1,173333	1	0,864968	0,897941
177	1,180000	1	0,867484	0,900145
178	1,186667	1	0,869964	0,902312
179	1,193333	1	0,872410	0,904445
180	1,200000	1	0,874821	0,906542
181	1,206667	1	0,877197	0,908605
182	1,213333	1	0,879540	0,910633
183	1,220000	1	0,881849	0,912628
184	1,226667	1	0,884125	0,914589
185	1,233333	1	0,886367	0,916517
186	1,240000	1	0,888577	0,918412
187	1,246667	1	0,890755	0,920276
188	1,253333	1	0,892900	0,922107
189	1,260000	1	0,895014	0,923907
190	1,266667	1	0,897096	0,925676
191	1,273333	1	0,899147	0,927414
192	1,280000	1	0,901168	0,929121
193	1,286667	1	0,903158	0,930799
194	1,293333	1	0,905117	0,932448
195	1,300000	1	0,907047	0,934067
~	~	~	~	~

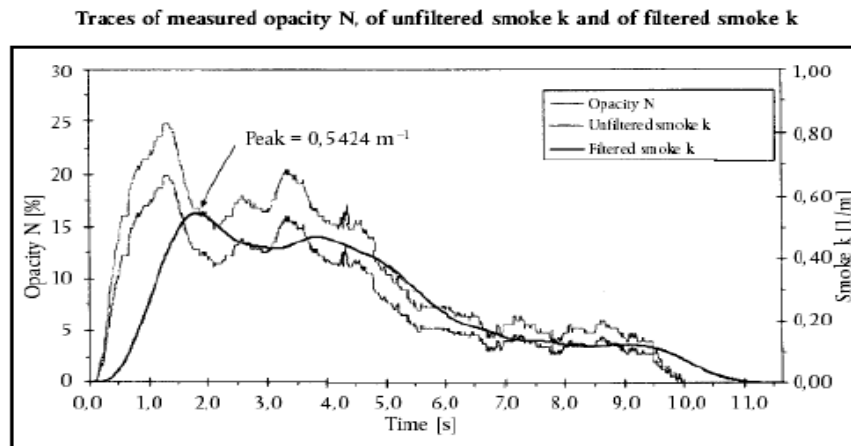
2.3. Calculation of the smoke Values

In the scheme below the general procedure of determining the final smoke value is presented.



In Figure b, the traces of the measured raw opacity signal, and of the unfiltered and filtered light absorption coefficients (k-value) of the first load step of an ELR-Test are shown, and the maximum value  $Y_{max1,A}$  (peak) of the filtered k trace is indicated. Correspondingly, Table C contains the numerical values of index i, time (sampling rate of 150 Hz), raw opacity, unfiltered k and filtered k. Filtering was conducted using the constants of the Bessel algorithm designed in Section 2.2 of this Annex. Due to the large amount of data, only those sections of the smoke trace around the beginning and the peak are tabled.

Figure b



The peak value ( $i = 272$ ) is calculated assuming the following data of Table C. All other individual smoke values are calculated in the same way. For starting the algorithm,  $S_{-1}$ ,  $S_{-2}$ ,  $Y_{-1}$  and  $Y_{-2}$  are set to zero.

$L_A$ (m)	0,430
Index i	272
N (%)	16,783
$S_{271}$ ( $m^{-1}$ )	0,427392
$S_{270}$ ( $m^{-1}$ )	0,427532
$Y_{271}$ ( $m^{-1}$ )	0,542383
$Y_{270}$ ( $m^{-1}$ )	0,542337

Calculation of the k-value (Chapter III, Appendix 1, Section 6.3.1):

$$k = - (1/0,430) \times \ln (1 - (16,783/100)) = 0,427252 \text{ m}^{-1}$$

This value corresponds to  $S_{272}$  in the following equation.

Calculation of Bessel averaged smoke (Chapter III, Appendix 1, Section 6.3.2):

In the following equation, the Bessel constants of the previous Section 2.2 are used. The actual unfiltered k-value, as calculated above, corresponds to  $S_{272}$  ( $S_i$ ).  $S_{271}$  ( $S_{i-1}$ ) and  $S_{270}$  ( $S_{i-2}$ ) are the two preceding unfiltered k-values,  $Y_{271}$  ( $Y_{i-1}$ ) and  $Y_{270}$  ( $Y_{i-2}$ ) are the two preceding filtered k-values.

$$Y_{272} = 0,542383 + 8,272777 \times 10^{-5} \times (0,427252 + 2 \times 0,427392 + 0,427532 - 4 \times 0,542337) + 0,968410 \times (0,542383 - 0,542337)$$

$$= 0,542389 \text{ m}^{-1}$$

This value corresponds to  $Y_{\max 1,A}$  in the following equation.

Calculation of the final smoke value (Chapter III, Appendix 1, Section 6.3.3):

From each smoke trace, the maximum filtered k-value is taken for the further calculation.

Assume the following values

Speed	$Y_{\max} \text{ (m}^{-1}\text{)}$		
	Cycle 1	Cycle 2	Cycle 3
A	0,5424	0,5435	0,5587
B	0,5596	0,5400	0,5389
C	0,4912	0,5207	0,5177

$$SV_A = (0,5424 + 0,5435 + 0,5587) / 3 = 0,5482 \text{ m}^{-1}$$

$$SV_B = (0,5596 + 0,5400 + 0,5389) / 3 = 0,5462 \text{ m}^{-1}$$

$$SV_C = (0,4912 + 0,5207 + 0,5177) / 3 = 0,5099 \text{ m}^{-1}$$

$$SV = (0,43 * 0,5482) + (0,56 * 0,5462) + (0,01 * 0,5099) = 0,5467 \text{ m}^{-1}$$

Cycle validation (Chapter III, Appendix 1, Section 3.4)

Before calculating SV, the cycle must be validated by calculating the relative standard deviations of the smoke of the three cycles for each speed.

Speed	Mean SV (m <sup>-1</sup> )	Absolute standard deviation (m <sup>-1</sup> )	Relative standard deviation (%)
A	0,5482	0,0091	1,7
B	0,5462	0,0116	2,1
C	0,5099	0,0162	3,2

In this example, the validation criteria of 15 % are met for each speed.



Table C

## Values of opacity N, unfiltered and filtered k-value at beginning of load step

Index i [-]	Time [s]	Opacity N [%]	unfiltered k-value [m <sup>-1</sup> ]	filtered k-value [m <sup>-1</sup> ]
-2	0,000000	0,000000	0,000000	0,000000
-1	0,000000	0,000000	0,000000	0,000000
0	0,000000	0,000000	0,000000	0,000000
1	0,006667	0,020000	0,000465	0,000000
2	0,013333	0,020000	0,000465	0,000000
3	0,020000	0,020000	0,000465	0,000000
4	0,026667	0,020000	0,000465	0,000001
5	0,033333	0,020000	0,000465	0,000002
6	0,040000	0,020000	0,000465	0,000002
7	0,046667	0,020000	0,000465	0,000003
8	0,053333	0,020000	0,000465	0,000004
9	0,060000	0,020000	0,000465	0,000005
10	0,066667	0,020000	0,000465	0,000006
11	0,073333	0,020000	0,000465	0,000008
12	0,080000	0,020000	0,000465	0,000009
13	0,086667	0,020000	0,000465	0,000011
14	0,093333	0,020000	0,000465	0,000012
15	0,100000	0,192000	0,004469	0,000014
16	0,106667	0,212000	0,004935	0,000018
17	0,113333	0,212000	0,004935	0,000022
18	0,120000	0,212000	0,004935	0,000028
19	0,126667	0,343000	0,007990	0,000036
20	0,133333	0,566000	0,013200	0,000047
21	0,140000	0,889000	0,020767	0,000061
22	0,146667	0,929000	0,021706	0,000082
23	0,153333	0,929000	0,021706	0,000109
24	0,160000	1,263000	0,029559	0,000143
25	0,166667	1,455000	0,034086	0,000185
26	0,173333	1,697000	0,039804	0,000237
27	0,180000	2,030000	0,047695	0,000301
28	0,186667	2,081000	0,048906	0,000378
29	0,193333	2,081000	0,048906	0,000469
30	0,200000	2,424000	0,057067	0,000573
31	0,206667	2,475000	0,058282	0,000693
32	0,213333	2,475000	0,058282	0,000827
33	0,220000	2,808000	0,066237	0,000977
34	0,226667	3,010000	0,071075	0,001144
35	0,233333	3,253000	0,076909	0,001328
36	0,240000	3,606000	0,085410	0,001533
37	0,246667	3,960000	0,093966	0,001758
38	0,253333	4,455000	0,105983	0,002007
39	0,260000	4,818000	0,114836	0,002283
40	0,266667	5,020000	0,119776	0,002587
~	~	~	~	~

Values of opacity N, unfiltered and filtered k-value around  $Y_{\max, \Delta}$  (= peak value, indicated in bold number)

Index i	Time	Opacity N	unfiltered k-value	filtered k-value
[-]	[s]	[%]	[m <sup>-1</sup> ]	[m <sup>-1</sup> ]
~	~	~	~	~
259	1,726667	17,182000	0,438429	0,538856
260	1,733333	16,949000	0,431896	0,539423
261	1,740000	16,788000	0,427392	0,539936
262	1,746667	16,798000	0,427671	0,540396
263	1,753333	16,788000	0,427392	0,540805
264	1,760000	16,798000	0,427671	0,541163
265	1,766667	16,798000	0,427671	0,541473
266	1,773333	16,788000	0,427392	0,541735
267	1,780000	16,788000	0,427392	0,541951
268	1,786667	16,798000	0,427671	0,542123
269	1,793333	16,798000	0,427671	0,542251
270	1,800000	16,793000	0,427532	0,542337
271	1,806667	16,788000	0,427392	0,542383
272	1,813333	16,783000	0,427252	<b>0,542389</b>
273	1,820000	16,780000	0,427168	0,542357
274	1,826667	16,798000	0,427671	0,542288
275	1,833333	16,778000	0,427112	0,542183
276	1,840000	16,808000	0,427951	0,542043
277	1,846667	16,768000	0,426833	0,541870
278	1,853333	16,010000	0,405750	0,541662
279	1,860000	16,010000	0,405750	0,541418
280	1,866667	16,000000	0,405473	0,541136
281	1,873333	16,010000	0,405750	0,540819
282	1,880000	16,000000	0,405473	0,540466
283	1,886667	16,010000	0,405750	0,540080
284	1,893333	16,394000	0,416406	0,539663
285	1,900000	16,394000	0,416406	0,539216
286	1,906667	16,404000	0,416685	0,538744
287	1,913333	16,394000	0,416406	0,538245
288	1,920000	16,394000	0,416406	0,537722
289	1,926667	16,384000	0,416128	0,537175
290	1,933333	16,010000	0,405750	0,536604
291	1,940000	16,010000	0,405750	0,536009
292	1,946667	16,000000	0,405473	0,535389
293	1,953333	16,010000	0,405750	0,534745
294	1,960000	16,212000	0,411349	0,534079
295	1,966667	16,394000	0,416406	0,533394
296	1,973333	16,394000	0,416406	0,532691
297	1,980000	16,192000	0,410794	0,531971
298	1,986667	16,000000	0,405473	0,531233
299	1,993333	16,000000	0,405473	0,530477
300	2,000000	16,000000	0,405473	0,529704

### 3. ETC TEST

#### 3.1. Gaseous emissions (diesel engine)

Assume the following test results for a PDP-CVS system

$V_0$ (m <sup>3</sup> /rev)	0,1776
$n_p$ (rev)	23 073
$p_b$ (kPa)	98,0
$p_1$ (kPa)	2,3
$T$ (K)	322,5
$T_a$ (K)	298
$H_a$ (g/kg)	12,8
$C_{NO_{xe}}$ (ppm)	53,7
$C_{NO_{xd}}$ (ppm)	0,4
$C_{CO_e}$ (ppm)	38,9
$C_{CO_d}$ (ppm)	1,0
$C_{H_{Ce}}$ (ppm)	9,00
$C_{H_{Cd}}$ (ppm)	3,02
$C_{CO_{2e}}$ (%)	0,723
$W_{act}$ (kWh)	62,72

Calculation of the diluted exhaust gas flow (Chapter III, Appendix 2, Section 4.1):

$$m_{ed} = 1,293 \times 0,1776 \times 23\ 073 \times (98,0 - 2,3) \times 273 / (101,3 \times 322,5) = 4\ 237,2 \text{ kg}$$

Calculation of the NOx correction factor (Chapter III, Appendix 1, Section 5.3):

$$K_{h,D} = \frac{1}{1 - 0,0182 \times (12,8 - 10,71) + 0,0045 \times (298 - 298)} = 1,039$$

Calculation of the background corrected concentrations (chapter III, Appendix 2, Section 5.4.1):

Assuming a diesel fuel of the composition  $C_{12}H_{18}O_{0,01}$

$$F_s = 100 \times \frac{1}{1 + 1,8 / 2 + 3,76 \times (1 + 1,8 / 4 - 0,01 / 2)} = 13,6$$

$$DF = \frac{13,6}{0,723} = 18,69$$

$$0,723 + (9,00 + 38,9) \times 10^{-4}$$

$$C_{\text{NOx}} = 53,7 - 0,4 \times (1 - (1/18,69)) = 53,3 \text{ ppm}$$

$$C_{\text{CO}} = 38,9 - 1,0 \times (1 - (1/18,69)) = 37,9 \text{ ppm}$$

$$C_{\text{HC}} = 9,00 - 3,02 \times (1 - (1/18,69)) = 6,14 \text{ ppm}$$

Calculation of the emissions mass flow (Chapter III, Appendix 2, Section 5.4):

$$m_{\text{NOx}} = 0,001587 \times 53,3 \times 1,039 \times 4\,237,2 = 372,391 \text{ g}$$

$$m_{\text{CO}} = 0,000966 \times 37,9 \times 4\,237,2 = 155,129 \text{ g}$$

$$m_{\text{HC}} = 0,000479 \times 6,14 \times 4\,237,2 = 12,462 \text{ g}$$

Calculation of the specific emissions (Chapter III, Appendix 2, Section 5.5):

$$\text{NOx} = 372,391 / 62,72 = 5,94 \text{ g/kWh}$$

$$\text{CO} = 155,129 / 62,72 = 2,47 \text{ g/kWh}$$

$$\text{HC} = 12,462 / 62,72 = 0,199 \text{ g/kWh}$$

### 3.2. Particulate emissions (diesel engine)

Assume the following test results for a PDP-CVS system with double dilution

$m_{\text{ed}}$ (kg)	4 237,2
$m_{\text{f}}$ (mg)	3,074
$m_{\text{set}}$ (kg)	2,159
$m_{\text{ssd}}$ (kg)	0,909
$m_{\text{f,d}}$ (mg)	0,341
$m_{\text{d}}$ (kg)	1,245
D	18,69
Wact (kWh)	62,72

Calculation of the mass emission (Chapter III, Appendix 2, Section 6.2.1):

$$m_{\text{sep}} = 2,159 - 0,909 = 1,250 \text{ kg}$$

$$m_{PT} = (3.074 / 1.250) \times (4237.2 / 1000) = 10.42g$$

Calculation of the background corrected mass emission

(Chapter III, Appendix 2, Section 6.2.1):

$$m_{PT} = [(3.074 / 1.250) - ((0.341 / 1.245) \times (1 - (1/18.69)))] \times (4237.2 / 1000) = 9.32g$$

Calculation of the specific emission (Chapter III, Appendix 2, Section 5.2):

$$M_{PT} = 10,42 / 62,72 = 0,166 \text{ g / kWh}$$

$$M_{PT} = 9,32 / 62,72 = 0,149 \text{ g / kWh, if background corrected.}$$

### 3.3. Gaseous emissions (CNG engine)

Assume the following test results for a PDP-CVS system with double dilution

$m_{ed}$ (kg)	4 237,2
$H_a$ (g/kg)	12,8
$C_{NOxe}$ (ppm)	17,2
$C_{NOxd}$ (ppm)	0,4
$C_{COe}$ (ppm)	44,3
$C_{COd}$ (ppm)	1,0
$C_{Hce}$ (ppm)	27,0
$C_{HCd}$ (ppm)	3,02
$C_{CH4e}$ (ppm)	18,0

$C_{CH4d}$ (ppm)	1,7
$C_{CO2e}$ (%)	0,723
$W_{act}$ (kWh)	62,72

Calculation of the NOx, correction factor (Chapter III, Appendix 1, Section 5.3):

$$k_{h,G} = 0.6272 + (44.030 \times 10^{-3} \times 12.8) - (0.862 \times 10^{-3} \times (12.8)^2) = 1.049$$

Calculation of the NMHC concentration (Annex III, Appendix 2, Section 4.3.1):

(a) GC method

$$C_{NMHC} = 27,0 - 18,0 = 9,0 \text{ ppm}$$

(b) NMC method

Assuming a methane efficiency of 0,04 and an ethane efficiency of 0,98 (see Chapter III, Appendix 5, Section 1.8.4)

$$C_{HC(w/cutter)} = 18 \times (0.98 - 0.04) + 27 \times (1 - 0.98) = 17.46$$

$$C_{NMHC} = \frac{27 \times (1 - 0.04) - 17.46}{(0.98 - 0.04)} = 9.0 \text{ ppm}$$

Calculation of the background corrected concentrations (Chapter III, Appendix 2, Section 5.4.1):

Assuming a  $G_{20}$  reference fuel (100 % methane) of the composition  $C_1H_4O_{0.01}$ :

$$Fs = 100 \times \frac{1}{1 + 4 / 2 + 3.76 \times (1 + 4 / 4 - 0.01 / 2)} = 9.5$$

$$DF = \frac{9.5}{0.723 + (27,0 + 44,3) \times 10^{-4}} = 13,01$$

For NMHC, the background concentration is the difference between  $C_{HCd}$  and  $C_{CH4d}$

$$C_{NOx} = 17,2 - 0,4 \times (1 - (1/13,01)) = 16,8 \text{ ppm}$$

$$C_{CO} = 44,3 - 1,0 \times (1 - (1/13,01)) = 43,4 \text{ ppm}$$

$$C_{NMHC} = 9.0 - 1,32 \times (1 - (1/13,01)) = 7,8 \text{ ppm}$$

$$C_{CH4} = 18,0 - 1,7 \times (1 - (1/13,01)) = 16,4 \text{ ppm}$$

Calculation of the emissions mass flow (Chapter III, Appendix 2, Section 5.4):

$$m_{\text{NO}_x} = 0,001588 \times 16,8 \times 1,049 \times 4\,237,2 = 118,581 \text{ g}$$

$$m_{\text{CO}} = 0,000967 \times 43,4 \times 4\,237,2 = 177,826 \text{ g}$$

$$m_{\text{NMHC}} = 0,000584 \times 7,8 \times 4\,237,2 = 19.301 \text{ g}$$

$$m_{\text{CH}_4} = 0,000553 \times 16,4 \times 4\,237,2 = 38,428 \text{ g}$$

Calculation of the specific emissions (Annex III, Appendix 2, Section 4.4):

$$\text{NO}_x = 118.581 / 62,72 = 1,89 \text{ g/kWh}$$

$$\text{CO} = 177.826 / 62,72 = 2,84 \text{ g/kWh}$$

$$\text{NMHC} = 19.301 / 62,72 = 0,308 \text{ g/kWh}$$

$$\text{CH}_4 = 38.428 / 62,72 = 0,613 \text{ g/kWh}$$

#### 4. $\lambda$ -SHIFT FACTOR ( $S_\lambda$ )

##### 4.1. Calculation of the $\lambda$ -shift factor ( $S_\lambda$ ) (1)

$$S_\lambda = \frac{2}{\left(1 - \frac{\text{inert \%}}{100}\right) \left(n + \frac{m}{4}\right) - \frac{\text{O}_2^*}{100}}$$

where:

$S_\lambda$  =  $\lambda$ -shift factor;

inert % = % by volume of inert gases in the fuel (i.e. N<sub>2</sub>, CO<sub>2</sub>, He, etc.);

O<sub>2</sub>\* = % by volume of original oxygen in the fuel;

(<sup>1</sup>) Stoichiometric Air/Fuel ratios of automotive fuels - SAE J1829, June 1987.  
 John B. Heywood, Internal combustion engine fundamentals, McGraw-Hill, 1988,  
 Chapter 3.4 'Combustion stoichiometry' (pp. 68 to 72).

n and m = refer to average C<sub>n</sub>H<sub>m</sub> representing the fuel hydrocarbons, i.e:

$$n = \frac{1 \times \left[ \frac{\text{CH}_4 \%}{100} \right] + 2 \times \left[ \frac{\text{C}_2 \%}{100} \right] + 3 \times \left[ \frac{\text{C}_3 \%}{100} \right] + 4 \times \left[ \frac{\text{C}_4 \%}{100} \right] + 5 \times \left[ \frac{\text{C}_5 \%}{100} \right] + \dots}{\frac{1 - \text{diluent \%}}{100}}$$

$$m = \frac{4 \times \left[ \frac{\text{CH}_4 \%}{100} \right] + 4 \times \left[ \frac{\text{C}_2\text{H}_4 \%}{100} \right] + 6 \times \left[ \frac{\text{C}_2\text{H}_6 \%}{100} \right] + \dots + 8 \times \left[ \frac{\text{C}_3\text{H}_8 \%}{100} \right] + \dots}{\frac{1 - \text{diluent \%}}{100}}$$

where:

CH<sub>4</sub> = % by volume of methane in the fuel;

C<sub>2</sub> = % by volume of all C<sub>2</sub> hydrocarbons (e.g. C<sub>2</sub>H<sub>6</sub>, C<sub>2</sub>H<sub>4</sub>, etc.) in the fuel;

C<sub>3</sub> = % by volume of all C<sub>3</sub> hydrocarbons (e.g. C<sub>3</sub>H<sub>8</sub>, C<sub>3</sub>H<sub>6</sub>, etc.) in the fuel;

C<sub>4</sub> = % by volume of all C<sub>4</sub> hydrocarbons (e.g. C<sub>4</sub>H<sub>10</sub>, C<sub>4</sub>H<sub>8</sub>, etc.) in the fuel

C<sub>5</sub> = % by volume of all C<sub>5</sub> hydrocarbons (e.g. C<sub>5</sub>H<sub>12</sub>, C<sub>5</sub>H<sub>10</sub>, etc.) in the fuel;

diluent = % by volume of dilution gases in the fuel (i.e. O<sub>2</sub>\*, N<sub>2</sub>, CO<sub>2</sub>, He etc.).

#### 4.2. Examples for the calculation of the λ-shift factor S<sub>λ</sub>



Example 1: G<sub>25</sub>: CH<sub>4</sub> = 86 %, N<sub>2</sub> = 14 % (by volume)

$$n = \frac{1 \times \left[ \frac{\text{CH}_4 \%}{100} \right] + 2 \times \left[ \frac{\text{C}_2 \%}{100} \right] + \dots}{\frac{1 - \text{diluent \%}}{100}} = \frac{1 \times 0,86}{1 - \frac{14}{100}} = \frac{0,86}{0,86} = 1$$

$$m = \frac{4 \times \left[ \frac{\text{CH}_4 \%}{100} \right] + 4 \times \left[ \frac{\text{C}_2\text{H}_4 \%}{100} \right] + \dots}{\frac{1 - \text{diluent \%}}{100}} = \frac{4 \times 0,86}{0,86} = 4$$

$$S_\lambda = \frac{2}{\left(1 - \frac{\text{inert \%}}{100}\right) \left(n + \frac{m}{4}\right) - \frac{\text{O}_2^*}{100}} = \frac{2}{\left(1 - \frac{14}{100}\right) \times \left(1 + \frac{4}{4}\right)} = 1,16$$

Example 2: GR: CH<sub>4</sub> = 87 %, C<sub>2</sub>H<sub>6</sub> = 13 % (by vol)

$$n = \frac{1 \times \left[ \frac{\text{CH}_4 \%}{100} \right] + 2 \times \left[ \frac{\text{C}_2 \%}{100} \right] + \dots}{\frac{1 - \text{diluent \%}}{100}} = \frac{1 \times 0,87 + 2 \times 0,13}{1 - \frac{0}{100}} = \frac{1,13}{1} = 1,13$$

$$m = \frac{4 \times \left[ \frac{\text{CH}_4 \%}{100} \right] + 4 \times \left[ \frac{\text{C}_2\text{H}_4 \%}{100} \right] + \dots}{\frac{1 - \text{diluent \%}}{100}} = \frac{4 \times 0,87 + 6 \times 0,13}{1} = 4,26$$

$$S_\lambda = \frac{2}{\left(1 - \frac{\text{inert \%}}{100}\right) \left(n + \frac{m}{4}\right) - \frac{\text{O}_2^*}{100}} = \frac{2}{\left(1 - \frac{0}{100}\right) \times \left(1,13 + \frac{4,26}{4}\right)} = 0,911$$

Example 3: USA: CH<sub>4</sub> = 89 %, C<sub>2</sub>H<sub>6</sub> = 4,5 %, C<sub>3</sub>H<sub>8</sub> = 2,3 %, C<sub>6</sub>H<sub>14</sub> = 0,2 %, O<sub>2</sub> = 0,6 %, N<sub>2</sub> = 4 %

$$n = \frac{1 \times \left[ \frac{\text{CH}_4\%}{100} \right] + 2 \times \left[ \frac{\text{C}_2\%}{100} \right] + \dots}{\frac{1 - \text{diluent \%}}{100}} = \frac{1 \times 0,89 + 2 \times 0,045 + 3 \times 0,023 + 4 \times 0,002}{1 - \frac{(0,64 + 4)}{100}} = 1,11$$

$$m = \frac{4 \times \left[ \frac{\text{CH}_4\%}{100} \right] + 4 \times \left[ \frac{\text{C}_2\text{H}_4\%}{100} \right] + 6 \times \left[ \frac{\text{C}_2\text{H}_6}{100} \right] + \dots + 8 \times \left[ \frac{\text{C}_3\text{H}_8}{100} \right]}{\frac{1 - \text{diluent \%}}{100}}$$

$$= \frac{4 \times 0,89 + 4 \times 0,045 + 8 \times 0,023 + 14 \times 0,002}{1 - \frac{0,6 + 4}{100}} = 4,24$$

$$S_\lambda = \frac{2}{\left(1 - \frac{\text{inert \%}}{100}\right) \left(n + \frac{m}{4}\right) - \frac{\text{O}_2^*}{100}} = \frac{2}{\left(1 - \frac{4}{100}\right) \times \left(1,11 + \frac{4,24}{4}\right) - \frac{0,6}{100}} = 0,96$$