# Chapter VI

## EXAMPLE OF CALCULATION PROCEDURE

1. ESC TEST

1.1. **Gaseous emissions**

The measurement data for the calculation of the individual mode results are shown below. In this example, CO and $NO_x$ are measured on a dry basis, HC on a wet basis. The HC concentration is given in propane equivalent (C3) and has to be multiplied by 3 to result in the C1 equivalent. The calculation procedure is identical for the other modes.

| P (kW) | $T_a$ (K) | $H_a$ (g/kg) | $G_{EXH}$ (kg) | $G_{AIRW}$ (kg) | $G_{FUEL}$ (kg) | HC (ppm) | CO (ppm) | $NO_x$ (ppm) |
|---|---|---|---|---|---|---|---|---|
| 82,9 | 294,8 | 7,81 | 563,38 | 545,29 | 18,09 | 6,3 | 41,2 | 495 |

*Calculation of the dry to wet correction factor $K_{W,r}$ (Annex III, Appendix 1, section 4.2):*

$$F_{FH} = \frac{1{,}969}{\left(1 + \frac{18{,}09}{545{,}29}\right)} = 1{,}9058 \quad \text{and} \quad K_{W2} = \frac{1{,}608 * 7{,}81}{1\,000 + (1{,}608 * 7{,}81)} = 0{,}0124$$

$$K_{W,r} = \left(1 - 1{,}9058 * \frac{18{,}09}{541{,}06}\right) - 0{,}0124 = 0{,}9239$$

*Calculation of the wet concentrations:*

$$CO = 41{,}2 * 0{,}9239 = 38{,}1 \text{ ppm}$$

$$NO_x = 495 * 0{,}9239 = 457 \text{ ppm}$$

*Calculation of the $NO_x$ humidity correction factor $K_{H,D}$ (Annex III, Appendix 1, section 4.3):*

$$A = 0{,}309 * 18{,}09/541{,}06 - 0{,}0266 = -0{,}0163$$

$$B = -0{,}209 * 18{,}09/541{,}06 + 0{,}00954 = 0{,}0026$$

$$K_{H,D} = \frac{1}{1 - 0{,}0163 * (7{,}81 - 10{,}71) + 0{,}0026 * (294{,}8 - 298)} = 0{,}9625$$

*Calculation of the emission mass flow rates (Annex III, Appendix 1, section 4.4):*

$$NO_x = 0{,}001587 * 457 * 0{,}9625 * 563{,}38 = 393{,}27 \text{ g/h}$$

$$CO = 0{,}000966 * 38{,}1 * 563{,}38 = 20{,}735 \text{ g/h}$$

$$HC = 0{,}000479 * 6{,}3 * 3 * 563{,}38 = 5{,}100 \text{ g/h}$$

*Calculation of the specific emissions (Annex III, Appendix 1, section 4.5):*

The following example calculation is given for CO; the calculation procedure is identical for the other components.

The emission mass flow rates of the individual modes are multiplied by the respective weighting factors, as indicated in Annex III, Appendix 1, section 2.7.1, and summed up to result in the mean emission mass flow rate over the cycle:

$$CO = (6,7 * 0,15) + (24,6 * 0,08) + (20,5 * 0,10) + (20,7 * 0,10) + (20,6 * 0,05) + (15,0 * 0,05) + (19,7 * 0,05) + (74,5 * 0,09) + (31,5 * 0,10) + (81,9 * 0,08) + (34,8 * 0,05) + (30,8 * 0,05) + (27,3 * 0,05)$$

$$= 30,91 \text{ g/h}$$

The engine power of the individual modes is multiplied by the respective weighting factors, as indicated in Annex III, Appendix 1, section 2.7.1, and summed up to result in the mean cycle power:

$$P(n) = (0,1 * 0,15) + (96,8 * 0,08) + (55,2 * 0,10) + (82,9 * 0,10) + (46,8 * 0,05) + (70,1 * 0,05) + (23,0 * 0,05) + (114,3 * 0,09) + (27,0 * 0,10) + (122,0 * 0,08) + (28,6 * 0,05) + (87,4 * 0,05) + (57,9 * 0,05)$$

$$= 60,006 \text{ kW}$$

$$\overline{CO} = \frac{30,91}{60,006} = 0,0515 \text{ g/kWh}$$

*Calculation of the specific $NO_x$ emission of the random point (Annex III, Appendix 1, section 4.6.1):*

Assume the following values have been determined on the random point:

$n_Z$ = 1 600 $min^{-1}$

$M_Z$ = 495 Nm

$NO_{x\,mass,Z}$ = 487,9 g/h (calculated according to the previous formulae)

$P(n)_Z$ = 83 kW

$NO_{x,Z}$ = 487,9/83 = 5,878 g/kWh

*Determination of the emission value from the test cycle (Annex III, Appendix 1, section 4.6.2):*

Assume the values of the four enveloping modes on the ESC to be as follows:

| $n_{RT}$ | $n_{SU}$ | $E_R$ | $E_S$ | $E_T$ | $E_U$ | $M_R$ | $M_S$ | $M_T$ | $M_U$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 368 | 1 785 | 5,943 | 5,565 | 5,889 | 4,973 | 515 | 460 | 681 | 610 |

$E_{TU} = 5,889 + (4,973 - 5,889) * (1\,600 - 1\,368) / (1\,785 - 1\,368) = 5,377$ g/kWh

$E_{RS} = 5,943 + (5,565 - 5,943) * (1\,600 - 1\,368) / (1\,785 - 1\,368) = 5,732$ g/kWh

$M_{TU} = 681 + (601 - 681) * (1\,600 - 1\,368) / (1\,785 - 1\,368) = 641,3$ Nm

$M_{RS} = 515 + (460 - 515) * (1\,600 - 1\,368) / (1\,785 - 1\,368) = 484,3$ Nm

$E_Z = 5,732 + (5,377 - 5,732) * (495 - 484,3) / (641,3 - 484,3) = 5,708$ g/kWh

*Comparison of the $NO_x$ emission values (Annex III, Appendix 1, section 4.6.3):*

$$NO_{x\,diff} = 100 * (5,878 - 5,708)/5,708 = 2,98\,\%$$

1.2. **Particulate emissions**

Particulate measurement is based on the principle of sampling the particulates over the complete cycle, but determining the sample and flow rates ($M_{SAM}$ and $G_{EDF}$) during the individual modes. The calculation of $G_{EDF}$ depends on the system used. In the following examples, a system with $CO_2$ measurement and carbon balance method and a system with flow measurement are used. When using a full flow dilution system, $G_{EDF}$ is directly measured by the CVS equipment.

*Calculation of $G_{EDF}$ (Annex III, Appendix 1, sections 5.2.3 and 5.2.4):*

Assume the following measurement data of mode 4. The calculation procedure is identical for the other modes.

| $G_{EXH}$ (kg/h) | $G_{FUEL}$ (kg/h) | $G_{DILW}$ (kg/h) | $G_{TOTW}$ (kg/h) | $CO_{2D}$ (%) | $CO_{2A}$ (%) |
|---|---|---|---|---|---|
| 334,02 | 10,76 | 5,4435 | 6,0 | 0,657 | 0,040 |

(a) carbon balance method

$$G_{EDFW} = \frac{206,5 * 10,76}{0,657 - 0,040} = 3\,601,2 \text{ kg/h}$$

(b) flow measurement method

$$q = \frac{6,0}{(6,0 - 5,4435)} = 10,78$$

$$G_{EDFW} = 334,02 * 10,78 = 3\,600,7 \text{ kg/h}$$

*Calculation of the mass flow rate (Annex III, Appendix 1, section 5.4):*

The $G_{EDFW}$ flow rates of the individual modes are multiplied by the respective weighting factors, as indicated in Annex III, Appendix 1, section 2.7.1, and summed up to result in the mean $G_{EDF}$ over the cycle. The total sample rate $M_{SAM}$ is summed up from the sample rates of the individual modes.

$\overline{G_{EDFW}}$ = (3 567 * 0,15) + (3 592 * 0,08) + (3 611 * 0,10) + (3 600 * 0,10) + (3 618 * 0,05) + (3 600 * 0,05) + (3 640 * 0,05) + (3 614 * 0,09) + (3 620 * 0,10) + (3 601 * 0,08) + (3 639 * 0,05) + (3 582 * 0,05) + (3 635 * 0,05)

= 3 604,6 kg/h

$M_{SAM}$ = 0,226 + 0,122 + 0,151 + 0,152 + 0,076 + 0,076 + 0,076 + 0,136 + 0,151 + 0,121 + 0,076 + 0,076 + 0,075

= 1,515 kg

Assume the particulate mass on the filters to be 2,5 mg, then

$$PT_{mass} = \frac{2,5}{1,515} * \frac{3\,604,6}{1\,000} = 5,948 \text{ g/h}$$

*Background correction (optional)*

Assume one background measurement with the following values. The calculation of the dilution factor DF is identical to section 3.1 of this Annex and not shown here.

$$M_d = 0,1 \text{ mg; } M_{DIL} = 1,5 \text{ kg}$$

Sum of DF = [(1-1/119,15) * 0,15] + [(1-1/8,89) * 0,08] + [(1-1/14,75) * 0,10] + [(1-1/10,10) * 0,10] + [(1-1/18,02) * 0,05] + [(1-1/12,33) * 0,05] + [(1-1/32,18) * 0,05] + [(1-1/6,94) * 0,09] + [(1-1/25,19) * 0,10] + [(1-1/6,12) * 0,08] + [(1-1/20,87) * 0,05] + [(1-1/8,77) * 0,05] + [(1-1/12,59) * 0,05]

= 0,923

$$PT_{mass} = \frac{2,5}{1,515} - \left(\frac{0,1}{1,5} * 0,923\right) * \frac{3\,604,6}{1\,000} = 5,726 \text{ g/h}$$

*Calculation of the specific emission (Annex III, Appendix 1, section 5.5):*

P(n) = (0,1 * 0,15) + (96,8 * 0,08) + (55,2 * 0,10) + (82,9 * 0,10) + (46,8 * 0,05) + (70,1 * 0,05) + (23,0 * 0,05) + (114,3 * 0,09) + (27,0 * 0,10) + (122,0 * 0,08) + (28,6 * 0,05) + (87,4 * 0,05) + (57,9 * 0,05)

= 60,006 kW

$\overline{PT}$ = $\frac{5,948}{60,006}$ = 0,099 g/kWh

if background corrected $\overline{PT}$ = $\frac{5,726}{60,006}$ = 0,095 g/kWh

*Calculation of the specific weighting factor (Annex III, Appendix 1, section 5.6):*

Assume the values calculated for mode 4 above, then

$$WF_{E,i} = \frac{0{,}152 * 3\ 604{,}6}{1{,}515 * 3\ 600{,}7} = 0{,}1004$$

This value is within the required value of 0,10 ± 0,003.

2. ELR TEST

Since Bessel filtering is a completely new averaging procedure in European exhaust legislation, an explanation of the Bessel filter, an example of the design of a Bessel algorithm, and an example of the calculation of the final smoke value is given below. The constants of the Bessel algorithm only depend on the design of the opacimeter and the sampling rate of the data acquisition system. It is recommended that the opacimeter manufacturer provide the final Bessel filter constants for different sampling rates and that the customer use these constants for designing the Bessel algorithm and for calculating the smoke values.

2.1. **General Remarks on the Bessel Filter**

Due to high frequency distortions, the raw opacity signal usually shows a highly scattered trace. To remove these high frequency distortions a Bessel filter is required for the ELR-Test. The Bessel filter itself is a recursive, second-order low-pass filter which guarantees the fastest signal rise without overshoot.

Assuming a real time raw exhaust plume in the exhaust tube, each opacimeter shows a delayed and differently measured opacity trace. The delay and the magnitude of the measured opacity trace is primarily dependent on the geometry of the measuring chamber of the opacimeter, including the exhaust sample lines, and on the time needed for processing the signal in the electronics of the opacimeter. The values that characterise these two effects are called the physical and the electrical response time which represent an individual filter for each type of opacimeter.

The goal of applying a Bessel filter is to guarantee a uniform overall filter characteristic of the whole opacimeter system, consisting of:

— physical response time of the opacimeter ($t_p$)

— electrical response time of the opacimeter ($t_e$)

— filter response time of the applied Bessel filter ($t_F$)

The resulting overall response time of the system $t_{Aver}$ is given by:

$$t_{Aver} = \sqrt{t_F{}^2 + t_p{}^2 + t_e{}^2}$$

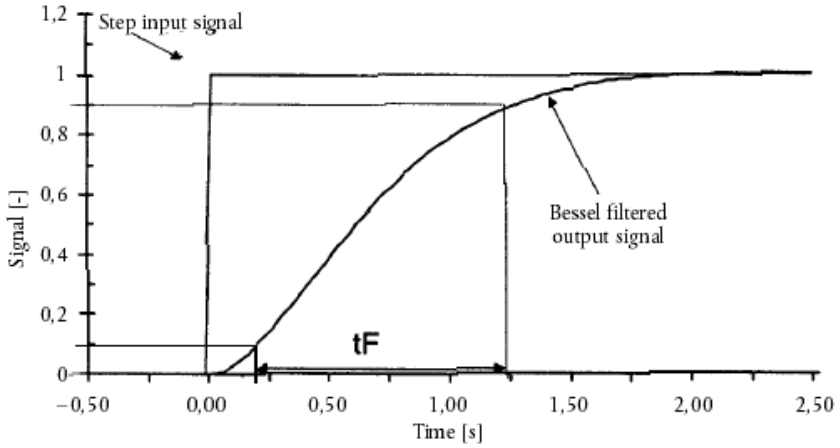
and must be equal for all kinds of opacimeters in order to give the same smoke value. Therefore, a Bessel filter has to be created in such a way, that the filter response time ($t_F$) together with the physical ($t_p$) and electrical response time ($t_e$) of the individual opacimeter must result in the required overall response time ($t_{Aver}$). Since $t_p$ and $t_e$ are given values for each individual opacimeter, and $t_{Aver}$ is defined to be 1,0 s in this Directive, $t_F$ can be calculated as follows:

$$t_F = \sqrt{t_{Aver}{}^2 + t_p{}^2 + t_e{}^2}$$

By definition, the filter response time $t_F$ is the rise time of a filtered output signal between 10% and 90% on a step input signal. Therefore the cut-off frequency of the Bessel filter has to be iterated in such a way, that the response time of the Bessel filter fits into the required rise time.

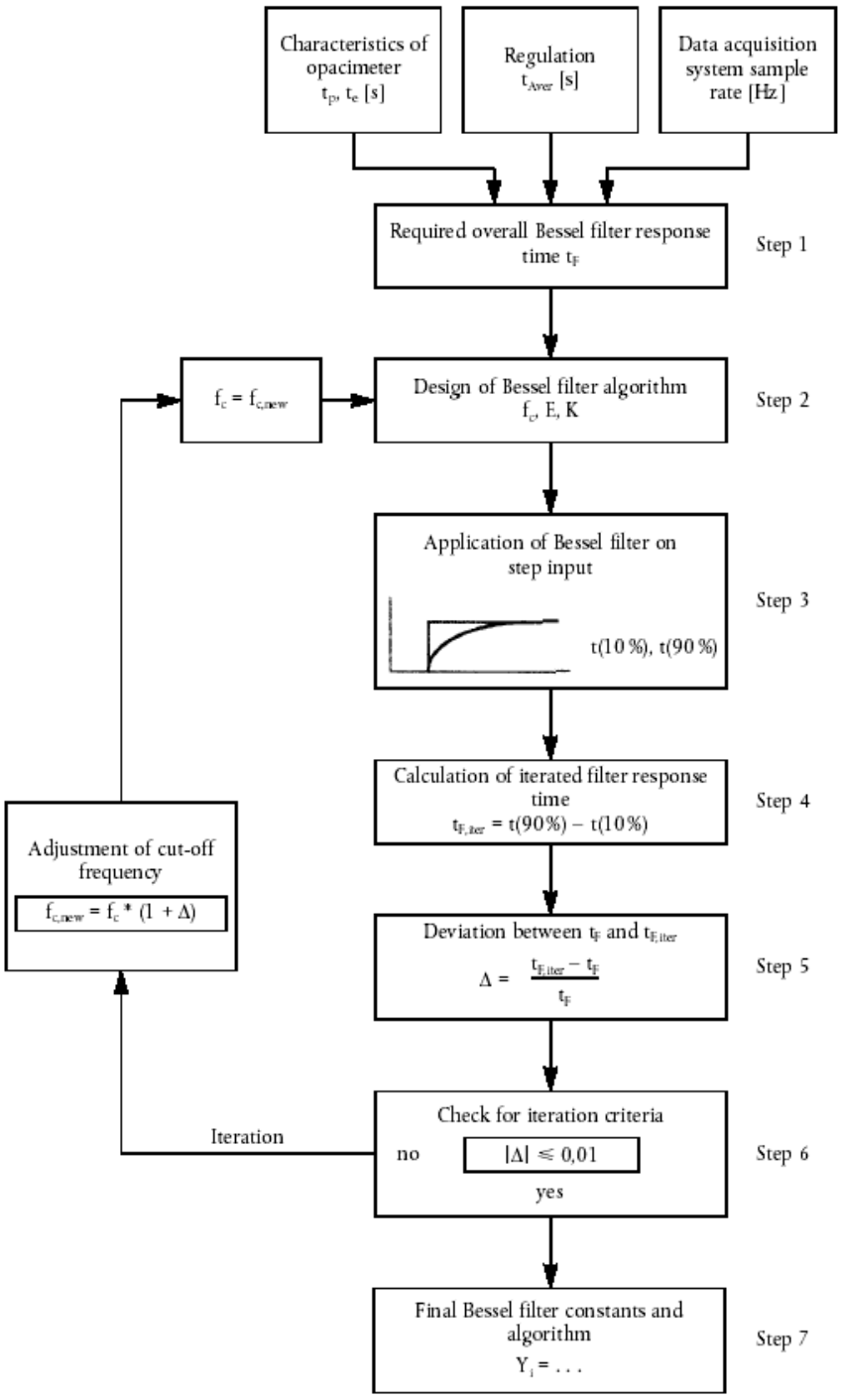
*Figure a*

**Traces of a step input signal and the filtered output signal**




In *Figure a*, the traces of a step input signal and Bessel filtered output signal as well as the response time of the Bessel filter ($t_F$) are shown.

Designing the final Bessel filter algorithm is a multi step process which requires several iteration cycles. The scheme of the iteration procedure is presented below.



2.2. **Calculation of the Bessel algorithm**

In this example a Bessel algorithm is designed in several steps according to the above iteration procedure which is based upon Annex III, Appendix 1, section 6.1.

For the opacimeter and the data acquisition system, the following characteristics are assumed:

— physical response time $t_p$ 0,15 s

— electrical response time $t_e$ 0,05 s

— overall response time $t_{Aver}$ 1,00 s (by definition in this Directive)

— sampling rate 150 Hz

*Step 1* *Required Bessel filter response time $t_F$:*

$$t_F = \sqrt{1^2 - (0{,}15^2 + 0{,}05^2)} = 0{,}987421 \text{ s}$$

*Step 2* *Estimation of cut-off frequency and calculation of Bessel constants E, K for first iteration:*

$f_c$ = 3,1415/(10 * 0,987421) = 0,318152 Hz

$\Delta t$ = 1/150 = 0,006667 s

$\Omega$ = 1/[tan (3,1415 * 0,006667 * 0,318152)] = 150,076644

$$E = \frac{1}{1 + 150{,}076644 * \sqrt{3 * 0{,}618034} + 0{,}618034 * 150{,}076644^2} = 7{,}07948\ E-5$$

$$K = 2 * 7{,}07948\ E-5 * (0{,}618034 * 150{,}076644^2 - 1) - 1 = 0{,}970783$$

This gives the Bessel algorithm:

$$Y_i = Y_{i-1} + 7{,}07948\ E-5 * (S_i + 2 * S_{i-1} + S_{i-2} - 4 * Y_{i-2}) + 0{,}970783 * (Y_{i-1} - Y_{i-2})$$

where $S_i$ represents the values of the step input signal (either '0' or '1') and $Y_i$ represents the filtered values of the output signal.

*Step 3* *Application of Bessel filter on step input:*

The Bessel filter response time $t_F$ is defined as the rise time of the filtered output signal between 10 % and 90 % on a step input signal. For determining the times of 10% ($t_{10}$) and 90 % ($t_{90}$) of the output signal, a Bessel filter has to be applied to a step input using the above values of $f_c$, E and K.

The index numbers, the time and the values of a step input signal and the resulting values of the filtered output signal for the first and the second iteration are shown in Table B. The points adjacent to $t_{10}$ and $t_{90}$ are marked in bold numbers.

In Table B, first iteration, the 10% value occurs between index number 30 and 31 and the 90 % value occurs between index number 191 and 192. For the calculation of $t_{F,iter}$ the exact $t_{10}$ and $t_{90}$ values are determined by linear interpolation between the adjacent measuring points, as follows:

$$t_{10} = t_{lower} + \Delta t * (0{,}1 - out_{lower})/(out_{upper} - out_{lower})$$

$$t_{90} = t_{lower} + \Delta t * (0{,}9 - out_{lower})/(out_{upper} - out_{lower})$$

where $out_{upper}$ and $out_{lower}$, respectively, are the adjacent points of the Bessel filtered output signal, and $t_{lower}$ is the time of the adjacent time point, as indicated in Table B.

$$t_{10} = 0{,}200000 + 0{,}006667 * (0{,}1 - 0{,}099208)/(0{,}104794 - 0{,}099208) = 0{,}200945 \text{ s}$$

$$t_{90} = 0{,}273333 + 0{,}006667 * (0{,}9 - 0{,}899147)/(0{,}901168 - 0{,}899147) = 1{,}276147 \text{ s}$$

*Step 4* *Filter response time of first iteration cycle:*

$$t_{F,iter} = 1{,}276147 - 0{,}200945 = 1{,}075202 \text{ s}$$

*Step 5* *Deviation between required and obtained filter response time of first iteration cycle:*

$$\Delta = (1{,}075202 - 0{,}987421)/0{,}987421 = 0{,}081641$$

*Step 6* *Checking the iteration criteria:*

$|\Delta| \leq 0{,}01$ is required. Since $0{,}081641 > 0{,}01$, the iteration criteria is not met and a further iteration cycle has to be started. For this iteration cycle, a new cut-off frequency is calculated from $f_c$ and $\Delta$ as follows:

$$f_{c,new} = 0{,}318152 * (1 + 0{,}081641) = 0{,}344126 \text{ Hz}$$

This new cut-off frequency is used in the second iteration cycle, starting at step 2 again. The iteration has to be repeated until the iteration criteria is met. The resulting values of the first and second iteration are summarised in table A.

*Table A*

**Values of the first and second iteration**

| Parameter | | 1. Iteration | 2. Iteration |
|---|---|---|---|
| $f_c$ | (Hz) | 0,318152 | 0,344126 |
| E | (–) | 7,07948 E-5 | 8,272777 E-5 |
| K | (–) | 0,970783 | 0,968410 |
| $t_{10}$ | (s) | 0,200945 | 0,185523 |
| $t_{90}$ | (s) | 1,276147 | 1,179562 |
| $t_{F,iter}$ | (s) | 1,075202 | 0,994039 |
| $\Delta$ | (–) | 0,081641 | 0,006657 |
| $f_{c,new}$ | (Hz) | 0,344126 | 0,346417 |

*Step 7* *Final Bessel algorithm:*

As soon as the iteration criteria has been met, the final Bessel filter constants and the final Bessel algorithm are calculated according to step 2. In this example, the iteration criteria has been met after the second iteration ($\Delta = 0{,}006657 \leq 0{,}01$). The final algorithm is then used for determining the averaged smoke values (see next section 2.3).

$$Y_i = Y_{i-1} + 8{,}272777\ E-5 * (S_i + 2 * S_{i-1} + S_{i-2} - 4 * Y_{i-2}) + 0{,}968410 * (Y_{i-1} - Y_{i-2})$$

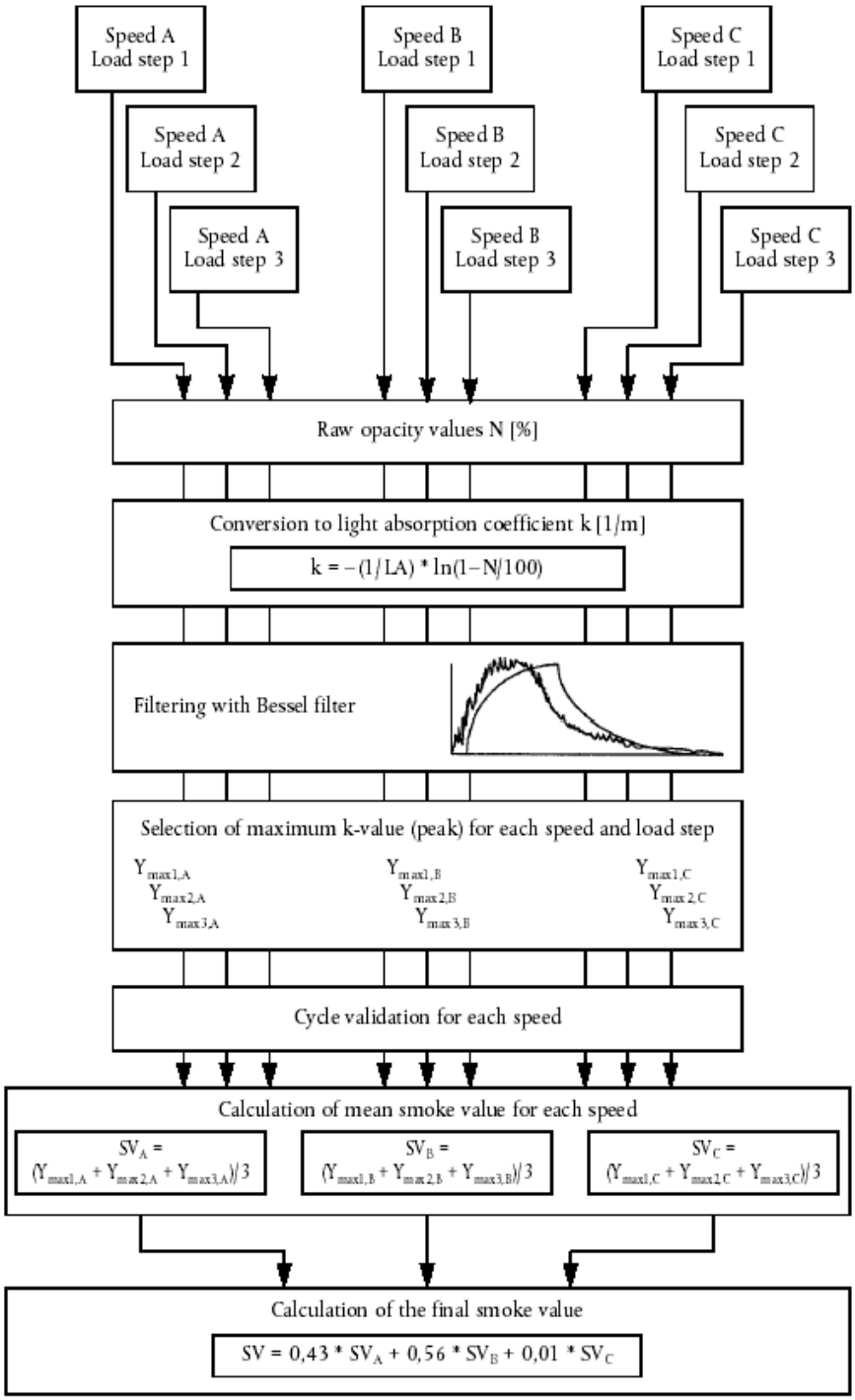
*Table B*

**Values of step input signal and Bessel filtered output signal for the first and second iteration cycle**

| Index i [−] | Time [s] | Step Input Signal $S_i$ [−] | Filtered Output Signal $Y_i$ [−] | |
|---|---|---|---|---|
| | | | 1. Iteration | 2. Iteration |
| −2 | −0,013333 | 0 | 0,000000 | 0,000000 |
| −1 | −0,006667 | 0 | 0,000000 | 0,000000 |
| 0 | 0,000000 | 1 | 0,000071 | 0,000083 |
| 1 | 0,006667 | 1 | 0,000352 | 0,000411 |
| 2 | 0,013333 | 1 | 0,000908 | 0,001060 |
| 3 | 0,020000 | 1 | 0,001731 | 0,002019 |
| 4 | 0,026667 | 1 | 0,002813 | 0,003278 |
| 5 | 0,033333 | 1 | 0,004145 | 0,004828 |
| ~ | ~ | ~ | ~ | ~ |
| 24 | 0,160000 | 1 | 0,067877 | 0,077876 |
| 25 | 0,166667 | 1 | 0,072816 | 0,083476 |
| 26 | 0,173333 | 1 | 0,077874 | 0,089205 |
| 27 | 0,180000 | 1 | 0,083047 | 0,095056 |
| 28 | 0,186667 | 1 | 0,088331 | 0,101024 |
| 29 | 0,193333 | 1 | 0,093719 | 0,107102 |
| 30 | 0,200000 | 1 | 0,099208 | 0,113286 |
| 31 | 0,206667 | 1 | 0,104794 | 0,119570 |
| 32 | 0,213333 | 1 | 0,110471 | 0,125949 |
| 33 | 0,220000 | 1 | 0,116236 | 0,132418 |
| 34 | 0,226667 | 1 | 0,122085 | 0,138972 |
| 35 | 0,233333 | 1 | 0,128013 | 0,145605 |
| 36 | 0,240000 | 1 | 0,134016 | 0,152314 |
| 37 | 0,246667 | 1 | 0,140091 | 0,159094 |
| ~ | ~ | ~ | ~ | ~ |
| 175 | 1,166667 | 1 | 0,862416 | 0,895701 |
| 176 | 1,173333 | 1 | 0,864968 | 0,897941 |
| 177 | 1,180000 | 1 | 0,867484 | 0,900145 |
| 178 | 1,186667 | 1 | 0,869964 | 0,902312 |
| 179 | 1,193333 | 1 | 0,872410 | 0,904445 |
| 180 | 1,200000 | 1 | 0,874821 | 0,906542 |
| 181 | 1,206667 | 1 | 0,877197 | 0,908605 |
| 182 | 1,213333 | 1 | 0,879540 | 0,910633 |
| 183 | 1,220000 | 1 | 0,881849 | 0,912628 |
| 184 | 1,226667 | 1 | 0,884125 | 0,914589 |
| 185 | 1,233333 | 1 | 0,886367 | 0,916517 |
| 186 | 1,240000 | 1 | 0,888577 | 0,918412 |
| 187 | 1,246667 | 1 | 0,890755 | 0,920276 |
| 188 | 1,253333 | 1 | 0,892900 | 0,922107 |
| 189 | 1,260000 | 1 | 0,895014 | 0,923907 |
| 190 | 1,266667 | 1 | 0,897096 | 0,925676 |
| 191 | 1,273333 | 1 | 0,899147 | 0,927414 |
| 192 | 1,280000 | 1 | 0,901168 | 0,929121 |
| 193 | 1,286667 | 1 | 0,903158 | 0,930799 |
| 194 | 1,293333 | 1 | 0,905117 | 0,932448 |
| 195 | 1,300000 | 1 | 0,907047 | 0,934067 |
| ~ | ~ | ~ | ~ | ~ |

2.3. **Calculation of the smoke Values**

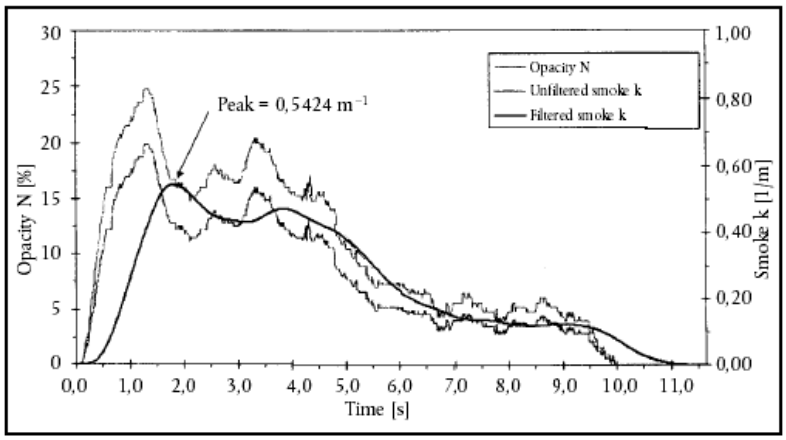
In the scheme below the general procedure of determining the final smoke value is presented.



In figure b, the traces of the measured raw opacity signal, and of the unfiltered and filtered light absorption coefficients (k-value) of the first load step of an ELR-Test are shown, and the maximum value $Y_{max1,A}$ (peak) of the filtered k trace is indicated. Correspondingly, table C contains the numerical values of index i, time (sampling rate of 150 Hz), raw opacity, unfiltered k and filtered k. Filtering was conducted using the constants of the Bessel algorithm designed in section 2.2 of this Annex. Due to the large amount of data, only those sections of the smoke trace around the beginning and the peak are tabled.

*Figure b*

**Traces of measured opacity N, of unfiltered smoke k and of filtered smoke k**




The peak value (i = 272) is calculated assuming the following data of table C. All other individual smoke values are calculated in the same way. For starting the algorithm, $S_{-1}$, $S_{-2}$, $Y_{-1}$ and $Y_{-2}$ are set to zero.

| | |
|---|---|
| $L_A$ (m) | 0,430 |
| Index i | 272 |
| N (%) | 16,783 |
| $S_{271}$ ($m^{-1}$) | 0,427392 |
| $S_{270}$ ($m^{-1}$) | 0,427532 |
| $Y_{271}$ ($m^{-1}$) | 0,542383 |
| $Y_{270}$ ($m^{-1}$) | 0,542337 |

*Calculation of the k-value (Annex III, Appendix 1, section 6.3.1):*

$$k = -\frac{1}{0,430} * \ln\left(1 - \frac{16,783}{100}\right) = 0,427252\ m^{-1}$$

This value corresponds to $S_{272}$ in the following equation.

*Calculation of Bessel averaged smoke (Annex III, Appendix 1, section 6.3.2):*

In the following equation, the Bessel constants of the previous Section 2.2 are used. The actual unfiltered k-value, as calculated above, corresponds to $S_{272}$ ($S_i$). $S_{271}$ ($S_{i-1}$) and $S_{270}$ ($S_{i-2}$) are the two preceding unfiltered k-values, $Y_{271}$ ($Y_{i-1}$) and $Y_{270}$ ($Y_{i-2}$) are the two preceding filtered k-values.

$Y_{272}$ = 0,542383 + 8,272777 E − 5 * (0,427252 + 2 * 0,427392 + 0,427532 − 4 * 0,542337) + 0,968410 * (0,542383 − 0,542337)

= 0,542389 $m^{-1}$

This value corresponds to $Y_{max1,A}$ in the following equation.

*Calculation of the final smoke value (Annex III, Appendix 1, section 6.3.3):*

From each smoke trace, the maximum filtered k-value is taken for the further calculation. Assume the following values

| Speed | $Y_{max}$ ($m^{-1}$) | | |
|---|---|---|---|
| | Cycle 1 | Cycle 2 | Cycle 3 |
| A | 0,5424 | 0,5435 | 0,5587 |
| B | 0,5596 | 0,5400 | 0,5389 |
| C | 0,4912 | 0,5207 | 0,5177 |

$SV_A$ = (0,5424 + 0,5435 + 0,5587) / 3 = 0,5482 $m^{-1}$

$SV_B$ = (0,5596 + 0,5400 + 0,5389) / 3 = 0,5462 $m^{-1}$

$SV_C$ = (0,4912 + 0,5207 + 0,5177) / 3 = 0,5099 $m^{-1}$

SV = (0,43 * 0,5482) + (0,56 * 0,5462) + (0,01 * 0,5099) = 0,5467 $m^{-1}$

*Cycle validation (Annex III, Appendix 1, section 3.4)*

Before calculating SV, the cycle must be validated by calculating the relative standard deviations of the smoke of the three cycles for each speed.

| Speed | Mean SV ($m^{-1}$) | absolute standard deviation ($m^{-1}$) | relative standard deviation (%) |
|---|---|---|---|
| A | 0,5482 | 0,0091 | 1,7 |
| B | 0,5462 | 0,0116 | 2,1 |
| C | 0,5099 | 0,0162 | 3,2 |

In this example, the validation criteria of 15% are met for each speed.

*Table C*

**Values of opacity N, unfiltered and filtered k-value at beginning of load step**

| Index i [-] | Time [s] | Opacity N [%] | unfiltered k-value [$m^{-1}$] | filtered k-value [$m^{-1}$] |
|---|---|---|---|---|
| -2 | 0,000000 | 0,000000 | 0,000000 | 0,000000 |
| -1 | 0,000000 | 0,000000 | 0,000000 | 0,000000 |
| 0 | 0,000000 | 0,000000 | 0,000000 | 0,000000 |
| 1 | 0,006667 | 0,020000 | 0,000465 | 0,000000 |
| 2 | 0,013333 | 0,020000 | 0,000465 | 0,000000 |
| 3 | 0,020000 | 0,020000 | 0,000465 | 0,000000 |
| 4 | 0,026667 | 0,020000 | 0,000465 | 0,000001 |
| 5 | 0,033333 | 0,020000 | 0,000465 | 0,000002 |
| 6 | 0,040000 | 0,020000 | 0,000465 | 0,000002 |
| 7 | 0,046667 | 0,020000 | 0,000465 | 0,000003 |
| 8 | 0,053333 | 0,020000 | 0,000465 | 0,000004 |
| 9 | 0,060000 | 0,020000 | 0,000465 | 0,000005 |
| 10 | 0,066667 | 0,020000 | 0,000465 | 0,000006 |
| 11 | 0,073333 | 0,020000 | 0,000465 | 0,000008 |
| 12 | 0,080000 | 0,020000 | 0,000465 | 0,000009 |
| 13 | 0,086667 | 0,020000 | 0,000465 | 0,000011 |
| 14 | 0,093333 | 0,020000 | 0,000465 | 0,000012 |
| 15 | 0,100000 | 0,192000 | 0,004469 | 0,000014 |
| 16 | 0,106667 | 0,212000 | 0,004935 | 0,000018 |
| 17 | 0,113333 | 0,212000 | 0,004935 | 0,000022 |
| 18 | 0,120000 | 0,212000 | 0,004935 | 0,000028 |
| 19 | 0,126667 | 0,343000 | 0,007990 | 0,000036 |
| 20 | 0,133333 | 0,566000 | 0,013200 | 0,000047 |
| 21 | 0,140000 | 0,889000 | 0,020767 | 0,000061 |
| 22 | 0,146667 | 0,929000 | 0,021706 | 0,000082 |
| 23 | 0,153333 | 0,929000 | 0,021706 | 0,000109 |
| 24 | 0,160000 | 1,263000 | 0,029559 | 0,000143 |
| 25 | 0,166667 | 1,455000 | 0,034086 | 0,000185 |
| 26 | 0,173333 | 1,697000 | 0,039804 | 0,000237 |
| 27 | 0,180000 | 2,030000 | 0,047695 | 0,000301 |
| 28 | 0,186667 | 2,081000 | 0,048906 | 0,000378 |
| 29 | 0,193333 | 2,081000 | 0,048906 | 0,000469 |
| 30 | 0,200000 | 2,424000 | 0,057067 | 0,000573 |
| 31 | 0,206667 | 2,475000 | 0,058282 | 0,000693 |
| 32 | 0,213333 | 2,475000 | 0,058282 | 0,000827 |
| 33 | 0,220000 | 2,808000 | 0,066237 | 0,000977 |
| 34 | 0,226667 | 3,010000 | 0,071075 | 0,001144 |
| 35 | 0,233333 | 3,253000 | 0,076909 | 0,001328 |
| 36 | 0,240000 | 3,606000 | 0,085410 | 0,001533 |
| 37 | 0,246667 | 3,960000 | 0,093966 | 0,001758 |
| 38 | 0,253333 | 4,455000 | 0,105983 | 0,002007 |
| 39 | 0,260000 | 4,818000 | 0,114836 | 0,002283 |
| 40 | 0,266667 | 5,020000 | 0,119776 | 0,002587 |
| ~ | ~ | ~ | ~ | ~ |

**Values of opacity N, unfiltered and filtered k-value around $Y_{max1,A}$ (≡ peak value, indicated in bold number)**

| Index i [−] | Time [s] | Opacity N [%] | unfiltered k-value [$m^{-1}$] | filtered k-value [$m^{-1}$] |
|---|---|---|---|---|
| ~ | ~ | ~ | ~ | ~ |
| 259 | 1,726667 | 17,182000 | 0,438429 | 0,538856 |
| 260 | 1,733333 | 16,949000 | 0,431896 | 0,539423 |
| 261 | 1,740000 | 16,788000 | 0,427392 | 0,539936 |
| 262 | 1,746667 | 16,798000 | 0,427671 | 0,540396 |
| 263 | 1,753333 | 16,788000 | 0,427392 | 0,540805 |
| 264 | 1,760000 | 16,798000 | 0,427671 | 0,541163 |
| 265 | 1,766667 | 16,798000 | 0,427671 | 0,541473 |
| 266 | 1,773333 | 16,788000 | 0,427392 | 0,541735 |
| 267 | 1,780000 | 16,788000 | 0,427392 | 0,541951 |
| 268 | 1,786667 | 16,798000 | 0,427671 | 0,542123 |
| 269 | 1,793333 | 16,798000 | 0,427671 | 0,542251 |
| 270 | 1,800000 | 16,793000 | 0,427532 | 0,542337 |
| 271 | 1,806667 | 16,788000 | 0,427392 | 0,542383 |
| 272 | 1,813333 | 16,783000 | 0,427252 | **0,542389** |
| 273 | 1,820000 | 16,780000 | 0,427168 | 0,542357 |
| 274 | 1,826667 | 16,798000 | 0,427671 | 0,542288 |
| 275 | 1,833333 | 16,778000 | 0,427112 | 0,542183 |
| 276 | 1,840000 | 16,808000 | 0,427951 | 0,542043 |
| 277 | 1,846667 | 16,768000 | 0,426833 | 0,541870 |
| 278 | 1,853333 | 16,010000 | 0,405750 | 0,541662 |
| 279 | 1,860000 | 16,010000 | 0,405750 | 0,541418 |
| 280 | 1,866667 | 16,000000 | 0,405473 | 0,541136 |
| 281 | 1,873333 | 16,010000 | 0,405750 | 0,540819 |
| 282 | 1,880000 | 16,000000 | 0,405473 | 0,540466 |
| 283 | 1,886667 | 16,010000 | 0,405750 | 0,540080 |
| 284 | 1,893333 | 16,394000 | 0,416406 | 0,539663 |
| 285 | 1,900000 | 16,394000 | 0,416406 | 0,539216 |
| 286 | 1,906667 | 16,404000 | 0,416685 | 0,538744 |
| 287 | 1,913333 | 16,394000 | 0,416406 | 0,538245 |
| 288 | 1,920000 | 16,394000 | 0,416406 | 0,537722 |
| 289 | 1,926667 | 16,384000 | 0,416128 | 0,537175 |
| 290 | 1,933333 | 16,010000 | 0,405750 | 0,536604 |
| 291 | 1,940000 | 16,010000 | 0,405750 | 0,536009 |
| 292 | 1,946667 | 16,000000 | 0,405473 | 0,535389 |
| 293 | 1,953333 | 16,010000 | 0,405750 | 0,534745 |
| 294 | 1,960000 | 16,212000 | 0,411349 | 0,534079 |
| 295 | 1,966667 | 16,394000 | 0,416406 | 0,533394 |
| 296 | 1,973333 | 16,394000 | 0,416406 | 0,532691 |
| 297 | 1,980000 | 16,192000 | 0,410794 | 0,531971 |
| 298 | 1,986667 | 16,000000 | 0,405473 | 0,531233 |
| 299 | 1,993333 | 16,000000 | 0,405473 | 0,530477 |
| 300 | 2,000000 | 16,000000 | 0,405473 | 0,529704 |
| ~ | ~ | ~ | ~ | ~ |

3. ETC TEST

3.1. **Gaseous emissions (Diesel Engine)**

Assume the following test results for a PDP-CVS system

| | |
|---|---|
| $V_0$ ($m^3$/rev) | 0,1776 |
| $N_p$ (rev) | 23 073 |
| $p_B$ (kPa) | 98,0 |
| $p_1$ (kPa) | 2,3 |
| T (K) | 322,5 |
| $H_a$ (g/kg) | 12,8 |
| $NO_{x\ conce}$ (ppm) | 53,7 |
| $NO_{x\ concd}$ (ppm) | 0,4 |
| $CO_{conce}$ (ppm) | 38,9 |
| $CO_{concd}$ (ppm) | 1,0 |
| $HC_{conce}$ (ppm) | 9,00 |
| $HC_{concd}$ (ppm) | 3,02 |
| $CO_{2,conce}$ (%) | 0,723 |
| $W_{act}$ (kWh) | 62,72 |

*Calculation of the diluted exhaust gas flow (Annex III, Appendix 2, section 4.1):*

$M_{TOTW}$ = 1,293 * 0,1776 * 23 073 * (98,0 − 2,3) * 273 / (101,3 * 322,5)

= 4 237,2 kg

*Calculation of the $NO_x$ correction factor (Annex III, Appendix 2, section 4.2):*

$$K_{H,D} = \frac{1}{1 - 0{,}0182 * (12{,}8 - 10{,}71)} = 1{,}039$$

*Calculation of the background corrected concentrations (Annex III, Appendix 2, section 4.3.1.1):*

Assuming a diesel fuel of the composition $C_1H_{1,8}$

$$F_S = 100 * \frac{1}{1 + (1{,}8/2) + \left[3{,}76 * \left(1 + (1{,}8/4)\right)\right]} = 13{,}6$$

$$DF = \frac{13{,}6}{0{,}723 + (9{,}00 + 38{,}9) * 10^{-4}} = 18{,}69$$

$$NO_{x\ conc} = 53{,}7 - 0{,}4 * \left(1 - (1/18{,}69)\right) = 53{,}3 \text{ ppm}$$

$$CO_{conc} = 38{,}9 - 1{,}0 * \left(1 - (1/18{,}69)\right) = 37{,}9 \text{ ppm}$$

$$HC_{conc} = 9{,}00 - 3{,}02 * \left(1 - (1/18{,}69)\right) = 6{,}14 \text{ ppm}$$

*Calculation of the emissions mass flow (Annex III, Appendix 2, section 4.3.1):*

$$NO_{x\ mass} = 0{,}001587 * 53{,}3 * 1{,}039 * 4\ 237{,}2 = 372{,}391 \text{ g}$$

$$CO_{mass} = 0{,}000966 * 37{,}9 * 4\ 237{,}2 = 155{,}129 \text{ g}$$

$$HC_{mass} = 0{,}000479 * 6{,}14 * 4\ 237{,}2 = 12{,}462 \text{ g}$$

*Calculation of the specific emissions (Annex III, Appendix 2, section 4.4):*

$$\overline{NO_x} = 372{,}391/62{,}72 = 5{,}94 \text{ g/kWh}$$

$$\overline{CO} = 155{,}129/62{,}72 = 2{,}47 \text{ g/kWh}$$

$$\overline{HC} = 12{,}462/62{,}72 = 0{,}199 \text{ g/kWh}$$

3.2. **Particulate emissions (Diesel Engine)**

Assume the following test results for a PDP-CVS system with double dilution

| | |
|---|---|
| $M_{TOTW}$ (kg) | 4 237,2 |
| $M_{f,p}$ (mg) | 3,030 |
| $M_{f,b}$ (mg) | 0,044 |
| $M_{TOT}$ (kg) | 2,159 |
| $M_{SEC}$ (kg) | 0,909 |
| $M_d$ (mg) | 0,341 |
| $M_{DIL}$ (kg) | 1,245 |
| DF | 18,69 |
| $W_{act}$ (kWh) | 62,72 |

*Calculation of the mass emission (Annex III, Appendix 2, section 5.1):*

$$M_f = 3{,}030 + 0{,}044 = 3{,}074 \text{ mg}$$

$$M_{SAM} = 2{,}159 - 0{,}909 = 1{,}250 \text{ kg}$$

$$PT_{mass} = \frac{3{,}074}{1{,}250} * \frac{4\ 237{,}2}{1\ 000} = 10{,}42 \text{ g}$$

*Calculation of the background corrected mass emission (Annex III, Appendix 2, section 5.1):*

$$PT_{mass} = \left[\frac{3{,}074}{1{,}250} - \left(\frac{0{,}341}{1{,}245} * \left(1 - \frac{1}{18{,}69}\right)\right)\right] * \frac{4\ 237{,}2}{1\ 000} = 9{,}32 \text{ g}$$

*Calculation of the specific emission (Annex III, Appendix 2, section 5.2):*

$$\overline{PT} = 10{,}42/62{,}72 = 0{,}166 \text{ g/kWh}$$

$$\overline{PT} = 9{,}32/62{,}72 = 0{,}149 \text{ g/kWh, if background corrected}$$

3.3. **Gaseous Emissions (CNG Engine)**

Assume the following test results for a PDP-CVS system with double dilution

| | |
|---|---|
| $M_{TOTW}$ (kg) | 4 237,2 |
| $H_a$ (g/kg) | 12,8 |
| $NO_{x\ conce}$ (ppm) | 17,2 |
| $NO_{x\ concd}$ (ppm) | 0,4 |
| $CO_{conce}$ (ppm) | 44,3 |
| $CO_{concd}$ (ppm) | 1,0 |
| $HC_{conce}$ (ppm) | 27,0 |
| $HC_{concd}$ (ppm) | 3,02 |
| $CH_{4\ conce}$ (ppm) | 18,0 |
| $CH_{4\ concd}$ (ppm) | 1,7 |
| $CO_{2,conce}$ (%) | 0,723 |
| $W_{act}$ (kWh) | 62,72 |

*Calculation of the $NO_x$ correction factor (Annex III, Appendix 2, section 4.2):*

$$K_{H,G} = \frac{1}{1 - 0{,}0329 * (12{,}8 - 10{,}71)} = 1{,}074$$

*Calculation of the NMHC concentration (Annex III, Appendix 2, section 4.3.1):*

(a) GC method

$$NMHC_{conce} = 27{,}0 - 18{,}0 = 9{,}0 \text{ ppm}$$

(b) NMC method

Assuming a methane efficiency of 0,04 and an ethane efficiency of 0,98 (see Annex III, Appendix 5, section 1.8.4)

$$NMHC_{conce} = \frac{27{,}0 * (1 - 0{,}04) - 18{,}0}{0{,}98 - 0{,}04} = 8{,}4 \text{ ppm}$$

*Calculation of the background corrected concentrations (Annex III, Appendix 2, section 4.3.1.1):*

Assuming a G20 reference fuel (100% methane) of the composition $C_1H_4$:

$$F_S = 100 * \frac{1}{1 + (4/2) + \left[3{,}76 * \left(1 + (4/4)\right)\right]} = 9{,}5$$

$$DF = \frac{9{,}5}{0{,}723 + (27{,}0 + 44{,}3) * 10^{-4}} = 13{,}01$$

For NMHC, the background concentration is the difference between $HC_{concd}$ and $CH_{4\ concd}$

$$NO_{x\ conc} = 17{,}2 - 0{,}4 * \left(1 - (1/13{,}01)\right) = 16{,}8 \text{ ppm}$$

$$CO_{conc} = 44{,}3 - 1{,}0 * \left(1 - (1/13{,}01)\right) = 43{,}4 \text{ ppm}$$

$$NMHC_{conc} = 8{,}4 - 1{,}32 * \left(1 - (1/13{,}01)\right) = 7{,}2 \text{ ppm}$$

$$CH_{4\ conc} = 18{,}0 - 1{,}7 * \left(1 - (1/13{,}01)\right) = 16{,}4 \text{ ppm}$$

*Calculation of the emissions mass flow (Annex III, Appendix 2, section 4.3.1):*

$$NO_{x\ mass} = 0{,}001587 * 16{,}8 * 1{,}074 * 4\ 237{,}2 = 121{,}330 \text{ g}$$

$$CO_{mass} = 0{,}000966 * 43{,}4 * 4\ 237{,}2 = 177{,}642 \text{ g}$$

$$NMHC_{mass} = 0{,}000502 * 7{,}2 * 4\ 237{,}2 = 15{,}315 \text{ g}$$

$$CH_{4\ mass} = 0{,}000554 * 16{,}4 * 4\ 237{,}2 = 38{,}498 \text{ g}$$

*Calculation of the specific emissions (Annex III, Appendix 2, section 4.4):*

$$\overline{NO_x} = 121{,}330/62{,}72 = 1{,}93 \text{ g/kWh}$$

$$\overline{CO} = 177{,}642/62{,}72 = 2{,}83 \text{ g/kWh}$$

$$\overline{NMHC} = 15{,}315/62{,}72 = 0{,}244 \text{ g/kWh}$$

$$\overline{CH_4} = 38{,}498/62{,}72 = 0{,}614 \text{ g/kWh}$$

4. λ-SHIFT FACTOR ($S_\lambda$)

4.1. **Calculation of the λ-shift factor ($S_\lambda$) [1]**

$$S_\lambda = \frac{2}{\left(1 - \frac{\text{inert \%}}{100}\right)\left(n + \frac{m}{4}\right) - \frac{O_2 *}{100}}$$

where:

$S_\lambda$ = λ-shift factor;

inert % = % by volume of inert gases in the fuel (i.e. $N_2$, $CO_2$, He, etc.);

$O_2$* = % by volume of original oxygen in the fuel;

[1] Stoichiometric Air/Fuel ratios of automotive fuels — SAE J1829, June 1987. John B. Heywood, *Internal combustion engine fundamentals*, McGraw-Hill, 1988, Chapter 3.4 'Combustion stoichiometry' (pp. 68 to 72).

n and m = refer to average $C_nH_m$ representing the fuel hydrocarbons, i.e:

$$n = \frac{1 \times \left[\frac{CH_4\%}{100}\right] + 2 \times \left[\frac{C_2\%}{100}\right] + 3 \times \left[\frac{C_3\%}{100}\right] + 4 \times \left[\frac{C_4\%}{100}\right] + 5 \times \left[\frac{C_5\%}{100}\right] + ..}{1 - \frac{\text{diluent \%}}{100}}$$

$$m = \frac{4 \times \left[\frac{CH_4\%}{100}\right] + 4 \times \left[\frac{C_2H_4\%}{100}\right] + 6 \times \left[\frac{C_2H_6\%}{100}\right] + ...8 \times \left[\frac{C_3H_8\%}{100}\right] + ..}{1 - \frac{\text{diluent \%}}{100}}$$

where:

$CH_4$ = % by volume of methane in the fuel;

$C_2$ = % by volume of all $C_2$ hydrocarbons (e.g.: $C_2H_6$, $C_2H_4$, etc.) in the fuel;

$C_3$ = % by volume of all $C_3$ hydrocarbons (e.g.: $C_3H_8$, $C_3H_6$, etc.) in the fuel;

$C_4$ = % by volume of all $C_4$ hydrocarbons (e.g.: $C_4H_{10}$, $C_4H_8$, etc.) in the fuel;

$C_5$ = % by volume of all $C_5$ hydrocarbons (e.g.: $C_5H_{12}$, $C_5H_{10}$, etc.) in the fuel;

diluent = % by volume of dilution gases in the fuel (i.e.: $O_2$*, $N_2$, $CO_2$, He, etc.).

4.2. **Examples for the calculation of the λ-shift factor $S_\lambda$:**

*Example 1:* $G_{25}$: $CH_4$ = 86%, $N_2$ = 14% (by volume)

$$n = \frac{1 \times \left[\frac{CH_4\%}{100}\right] + 2 \times \left[\frac{C_2\%}{100}\right] + ..}{1 - \frac{\text{diluent \%}}{100}} = \frac{1 \times 0{,}86}{1 - \frac{14}{100}} = \frac{0{,}86}{0{,}86} = 1$$

$$m = \frac{4 \times \left[\frac{CH_4\%}{100}\right] + 4 \times \left[\frac{C_2H_4\%}{100}\right] + ..}{1 - \frac{\text{diluent \%}}{100}} = \frac{4 \times 0{,}86}{0{,}86} = 4$$

$$S_\lambda = \frac{2}{\left(1 - \frac{\text{inert \%}}{100}\right)\left(n + \frac{m}{4}\right) - \frac{O_2\ *}{100}} = \frac{2}{\left(1 - \frac{14}{100}\right) \times \left(1 + \frac{4}{4}\right)} = 1{,}16$$

*Example 2:* $G_{xy}$: $CH_4$ = 87 %, $C_2H_6$ = 13% (by volume)

$$n = \frac{1 \times \left[\frac{CH_4\%}{100}\right] + 2 \times \left[\frac{C_2\%}{100}\right] + ..}{1 - \frac{\text{diluent \%}}{100}} = \frac{1 \times 0{,}87 + 2 \times 0{,}13}{1 - \frac{0}{100}} = \frac{1{,}13}{1} = 1{,}13$$

$$m = \frac{4 \times \left[\frac{CH_4\%}{100}\right] + 6 \times \left[\frac{C_2H_6\%}{100}\right] + ..}{1 - \frac{\text{diluent \%}}{100}} = \frac{4 \times 0{,}87 + 6 \times 0{,}13}{1} = 4{,}26$$

$$S_\lambda = \frac{2}{\left(1 - \frac{\text{inert \%}}{100}\right)\left(n + \frac{m}{4}\right) - \frac{O_2\ *}{100}} = \frac{2}{\left(1 - \frac{0}{100}\right) \times \left(1{,}13 + \frac{4{,}26}{4}\right)} = 0{,}911$$

*Example 3:* USA: $CH_4$ = 89 %, $C_2H_6$ = 4,5 %, $C_3H_8$ = 2,3 %, $C_6H_{14}$ = 0,2 %, $O_2$ = 0,6 %, $N_2$ = 4 %

$$n = \frac{1 \times \left[\frac{CH_4\%}{100}\right] + 2 \times \left[\frac{C_2\%}{100}\right] + ..}{1 - \frac{\text{diluent \%}}{100}} = \frac{1 \times 0{,}89 + 2 \times 0{,}045 + 3 \times 0{,}023 + 4 \times 0{,}002}{1 - \frac{(0{,}64 + 4)}{100}} = 1{,}11$$

$$m = \frac{4 \times \left[\frac{CH_4\%}{100}\right] + 4 \times \left[\frac{C_2H_4\%}{100}\right] + 6 \times \left[\frac{C_2H_6\%}{100}\right] + .. + 8 \times \left[\frac{C_3H_8\%}{100}\right]}{1 - \frac{\text{diluent \%}}{100}}$$

$$= \frac{4 \times 0{,}89 + 4 \times 0{,}045 + 8 \times 0{,}023 + 14 \times 0{,}002}{1 - \frac{0{,}6 + 4}{100}} = 4{,}24$$

$$S_\lambda = \frac{2}{\left(1 - \frac{\text{inert \%}}{100}\right)\left(n + \frac{m}{4}\right) - \frac{O_2{}^*}{100}} = \frac{2}{\left(1 - \frac{4}{100}\right) \times \left(1{,}11 + \frac{4{,}24}{4}\right) - \frac{0{,}6}{100}} = 0{,}96$$